MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL LXXIV. DECEMBER 12, 1913. NO. 2

Major E. H. HILLS, C.M.G., F.R.S., PRESIDENT, in the Chair.

F. C. H. Carpenter, M.A., B.Sc., University Observatory, Durham;
Douglas F. Horsefield, St. Silas' Vicarage, Bristol;
Harold Spencer Jones, B.A., B.Sc., Royal Observatory, Greenwich, S.E.; and
Herbert George Robins, F.G.S., F.R.G.S., F.R.M.S., F.R. Met. Soc., Bulawayo, Southern Rhodesia, South Africa,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Richard Henry Bulkeley, J.P., Wallerawang, New South Wales, Australia (proposed by W. F. Gale);
Richard Thomas Cullen, Royal Observatory, Greenwich, S.E. (proposed by F. W. Dyson);
R. Jarry-Desloges, Director of the Observatory, Sétif, Algeria (proposed by A. de la Baume Pluvinel);
Robert John Pocock, B.A., B.Sc., Queen's College, Oxford (proposed by H. H. Turner);
William Thow, M.Inst.C.E., Inglewood, Warrawee, New South Wales, Australia (proposed by W. F. Gale);

Lloyd A. H. Warren, M.A., Ph.D., Assistant Professor of Mathematics and Astronomy in the University of Manitoba, Winnipeg, Canada (proposed by F. R. Moulton); and
William Moody Witchell, Royal Observatory, Greenwich, and 36 Sandbourne Road, Brockley, S.E. (proposed by F. W. Dyson).

Dr. J. W. Capstick, Mr. H. P. Hollis, and Mr. Richard Inwards were appointed Auditors of the Treasurer's Accounts for 1913.

Fifty-four presents were announced as having been received since the last meeting, including, amongst others:—
Tycho Brahe, Opera Omnia, edidit J. L. E. Dreyer, Tomus I., presented by Dr. Dreyer; Birkeland (Kr.), The Norwegian Aurora Polaris Expedition, 1902–3; Cause of Magnetic Storms and Origin of Terrestrial Magnetism, 2 vols., presented by the author; Perth Observatory, Western Australia, Astrographic Catalogue, Perth Section, vols. 2, 3, presented by the Observatory.

Retrograde Satellite Orbits. By John Jackson, M.A., B.Sc., B.A., Mackinnon Student of the Royal Society.

I. *General Discussion.*

Until Hill published his *Researches in the Lunar Theory* practically all our knowledge of the possible solutions of the problem of three bodies was derived from the discussion of the motions of such bodies as were found in the solar system. The body whose motion departed most from the Keplerian ellipse was the moon, and when inquiries came to be made into more complex motions than existed in our system, it was natural that the question of remote satellites should be first considered, especially as the theory of the moon's motion was not altogether satisfactory. In his *Researches* Hill gave the "variational curves" for eleven different values of the relative mean motion of the satellite and the primary, ranging from the case of our moon to that of a satellite having 1·78 lunations in the period of the primary. The motion in all the cases actually treated by Hill was direct. Darwin by laborious arithmetical methods obtained a considerable number of "periodic orbits," but only one refers to a retrograde satellite, and the results it indicated seem to have been ignored. Poincaré, Brown, and others have considered by analytical processes various possible types of periodic motion, but never with respect to retrograde satellites. The discoveries of Phœbe and J VIII moving in retrograde orbits at great distances from their respective

primaries seem to warrant a discussion of the peculiarities of retrograde satellite motion in general.

We will first consider the retrograde periodic orbit treated by Darwin. With the usual notation for rotating axes Jacobi's integral of relative energy may be written in the form

$$V^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 2\Omega - C,$$

where

$$2\Omega = m\left(\frac{2}{r} + r^2\right) + m'\left(\frac{2}{r'} + r'^2\right).$$

V is the velocity relative to the rotating axes, and when direct and retrograde satellites with equal velocities in space are considered V will be larger for the retrograde motion. Now, the perturbative force which a satellite suffers depends for its order of magnitude on the distance from the primary; and so, if we are to consider direct and retrograde satellites for which the perturbative forces are about equal, we shall require to take satellites about equally distant from the primary, and therefore with about equal mean motions. For two such satellites 2Ω will have nearly the same value, but V will be larger and C smaller for the retrograde satellite. Darwin throughout his work used C as a variable parameter, and when he calculated his one retrograde satellite * he took as the value of C one of the larger values which he had used in connection with direct satellites. With such a value of C, 2Ω takes a rather large value and the satellite is comparatively near its primary. In this work Darwin always projected his infinitesimal body from a point on the line joining the sun to Jove (a large planet $= \frac{1}{10}$ sun), beyond Jove, first fixing the distance and then determining V in accordance with the value of C chosen. Using the distance from the sun to Jove as unity, he first guessed the distance from Jove as ·23. After this he tried ·22, ·20, and then, apparently recognising that he had been wide of the mark, he tried ·05. His final determination is ·0836. We are not surprised to read:† "The period is short compared with that of most of the orbits previously traced. The orbit is very stable . . ." The ratio of the sidereal motion to that of Jove was found to be 12·21. A comparison with Hill's work shows that the greatest and least radii vectores would not differ by more than 1·5 per cent. It does not seem to have been recognised even by Darwin that the series of non-periodic orbits which he traced was gradually leading him to a circle. As a matter of fact, the periodic orbit, in the search for which this work was undertaken, is nearer to a circle than any of the orbits actually traced. It is unfortunate that we have no remote retrograde satellite in Darwin's work, and the chief interest of this investigation must be considered to lie in the non-periodic orbits and not in the periodic orbit in the search for which they were constructed.

It may be interesting to find the differences to be used in the value of C for direct and retrograde satellites. Let C, V refer to

* *Scientific Papers*, vol. iv. pp. 155–157. † *L.c.*, p. 178.

a direct satellite, C_1, V_1 to a retrograde one moving in an approximately coincident orbit. The values of 2Ω will be approximately equal, so that we may write $C_1 + V_1^2 = C + V^2$. Now, for a circular orbit at distance r we have, in Darwin's units, $V = \frac{1}{r} - nr$, $V_1 = \frac{1}{r} + nr$, where n is the angular velocity of the rotating axes. $\therefore V_1^2 - V^2 = 4n$. In Darwin's work $n = \sqrt{11}$, so that $4n = 13{\cdot}27$. Fig. 1, founded on Darwin's orbits, gives direct satellites for $C = 40$, $40{\cdot}25$, $40{\cdot}5$, and the retrograde satellite for $C = 40$. A much more interesting retrograde orbit would have been obtained by using $C = 27$. It may be noted here that for C less than 33 Jacobi's integral fails to indicate any region from which the satellite is excluded.

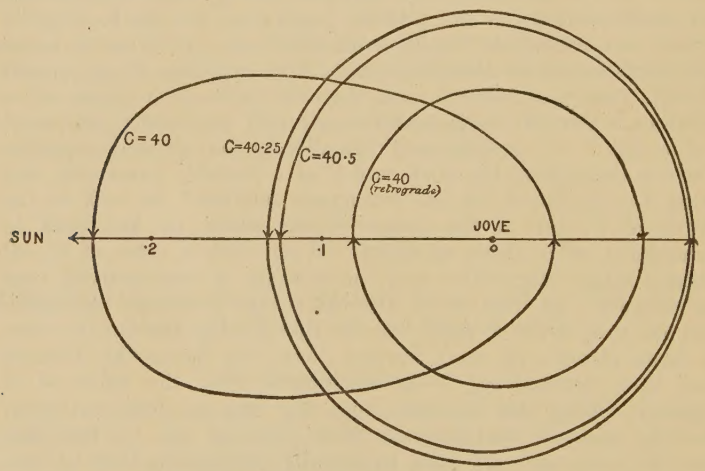


FIG. 1.—Periodic Orbits (after Darwin).

The equation $V^2 = 2\Omega - C$ has been used by Hill* to prove that the moon must remain for all time within a certain distance of the earth, only the eccentricity of the solar orbit being neglected. The proof is as follows:—Surfaces $2\Omega - C = 0$ are drawn for different values of C, and it is found that for C sufficiently large the surfaces have closed branches surrounding the planet. In the case of our moon the constant C is large enough for the corresponding surface $2\Omega - C = 0$ to be closed. As the value of $2\Omega - C$ is positive inside the surface and negative outside, and as $V^2 = 2\Omega - C$ must be positive for real motion, the moon must always remain within this closed surface. Similar results for all the other direct satellites in the solar system may also be obtained, but when we come to remote retrograde satellites no such conclusion can be simply drawn. It is stated by Moulton† that

* *Mathematical Works*, vol. i. pp. 296–302.
† *Astrophysical Journal*, vol. xxii. pp. 177–178.

Phœbe is safely held by Saturn, but the margin is much less than he indicated. In his units the smallest value of C which gives a closed surface for $2\Omega - C = 0$ is 3·0180, and for Phœbe he deduced the value 3·0626. Before discussing these figures it should be noted that the greater part arises from two terms which are nearly constant in the neighbourhood of the planet. 2Ω contains the terms $m'(r'^2 + 2/r')$ where m' is the mass of the sun in terms of the sum of the masses of sun and Saturn, and r' is the distance of the satellite from the sun in terms of the distance of Saturn from the sun. Clearly for quite a large region round Saturn these terms have a value nearly equal to 3. It would therefore be convenient to use C – 3, or a multiple of it, in discussing the cases of different satellites. With this criterion Moulton's figures show a very considerable margin of safety for Phœbe. But Moulton seems to have neglected V^2 in the calculation of C;* *i.e.* he seems to have calculated C from 2Ω instead of $2\Omega - V^2$. For retrograde motion V^2 is not small, and allowing for it the constant for Phœbe is 3·0270. This figure still indicates Phœbe to be a permanent member of the Saturnian system, but when we consider the eccentricity of ·056 in the orbit of Saturn it is questionable if Jacobi's integral can furnish a reliable proof that Phœbe will always remain in the neighbourhood of Saturn.

When we come to consider J VIII we find that Jacobi's integral completely fails to indicate any limit to the radius vector from Jupiter. In Moulton's units $2\Omega - C = 0$ gives closed surfaces round Jupiter for C greater than 3·0397, and closed surfaces round Jupiter and sun for C greater than 3·0384. But for J VIII the constant is 3·0200,* so that so far as Jacobi's integral can give information J VIII might not only escape from the neighbourhood of Jupiter but even from the solar system. It will be shown later from different considerations that we need not expect such an escape.

It may be advisable here to refer to some misconceptions which have arisen in connection with J VIII. When a retrograde satellite was first announced it was stated that the assumption of direct motion would lead to so long a period (about five and a half years) that the body would be outside Hill's limit for stable satellites. On the other hand, the assumption that the revolution was retrograde produced an orbit lying within the closed folds of the curves of constant velocity.† This statement seems to have led some to the conclusion that J VIII would always remain within these closed folds. J VIII is at present undoubtedly within the closed region shaded vertically in fig. 2. But the constant of relative energy for J VIII gives the curve C_3, and Jacobi's integral merely proves that the satellite is excluded from the region shaded horizontally. The nature of the region from which J VIII is

* See p. 78.

† *Cf.* reports of the meetings of the R.A.S. in the *Observatory* for 1908 April, July.

excluded is interesting. It proves that whatever may happen to J VIII it will not become a minor planet with the same mean motion as Jupiter.

It might seem that we are propounding a theory of lesser stability for retrograde than for direct satellites. We may roughly define one satellite to be more stable than another if it cannot recede so far from the primary as the other. For sufficiently large values of C, Jacobi's integral proves that a satellite must always remain within a certain distance of its primary, and this distance decreases as C increases. Moreover, we have seen C is

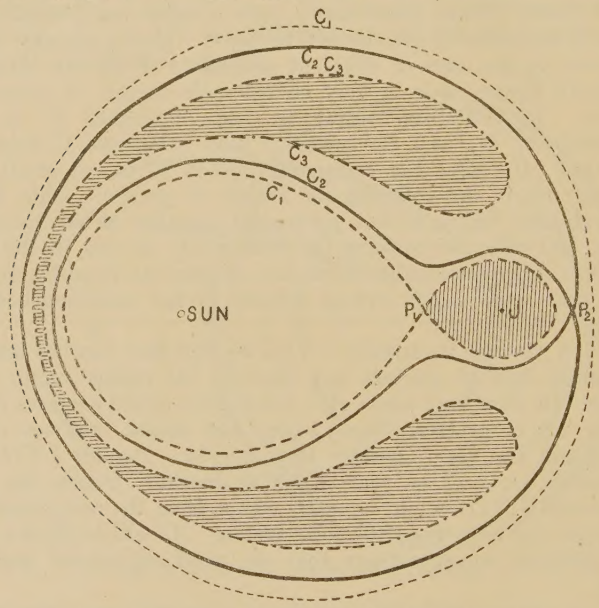


FIG. 2.—Intersections of surfaces $2\Omega - C = 0$, with ecliptic. The constants associated with the curves C_1, C_2, C_3 are 3·0397, 3·0384, 3·0200.

[These curves are only roughly drawn so as to show their general nature. For accurate curves see Plates 6, 7 of *M.N.*, lxxi. p. 444. To get Moulton's units multiply constants there given by $\frac{1048}{1049}$.]

greater for a direct than for a retrograde satellite equally remote from the primary. But Jacobi's integral does not determine the maximum distance to which a satellite can recede; it only proves the maximum distance to be less than a certain amount. For a satellite to recede to the distance indicated by Jacobi's integral its velocity relative to the rotating axes would require to become zero. This could quite conceivably take place in the case of direct satellites, and so Jacobi's integral seems to be a useful criterion for such bodies; in fact, it has proved all direct satellites in the solar system to be permanently attached to their primaries. But a retrograde satellite cannot be at rest relative to the rotating axes, and therefore cannot recede to such a distance as Jacobi's integral

gives as the maximum, unless the direction of revolution be altered. Until we can prove that retrograde satellites can become direct it is absurd to attempt employing Jacobi's integral to prove their instability. Jacobi's integral, indeed, cannot prove instability for any satellite; it can prove stability for some, it can prove nothing for others. In the solar system it proves all stable except J VIII; for J VIII it proves nothing (except that it will not become a minor planet with same mean motion as Jupiter).

We now proceed to consider in general terms the relative perturbations suffered by direct and retrograde satellites as the result of solar attraction. It may be as well to state definitely that the following argument makes no claim to cover all possible types of satellite motion. It is supposed throughout, as indeed has been done in the previous paragraph, that the satellite is moving more or less nearly perpendicular to the radius vector from the planet. Should a satellite move nearly along the radius vector the distinction between direct and retrograde motion might disappear. In the following discussions the distinction between direct and retrograde satellites is supposed to be complete. The treatment applies therefore only to orbits such as actually exist in the solar system, so far at least as general shape is concerned, although cases are considered when the satellite is much more remote than any known satellite. The orbits need not be of an exactly periodic nature like those considered by Darwin, or the "variational curves" of Hill; but the arguments can only be applied to bodies which circulate round planets. Finally, we qualify our arguments by assuming the orbit of the satellite not to be so highly inclined to the ecliptic as to make the terms direct and retrograde inconvenient or misleading.

So far from it being the case that the orbits of direct satellites are more stable than retrograde ones, as might have been supposed from Jacobi's integral, the exact opposite appears to be the case.* The principal terms in the differential equations for satellite perturbations contain as argument of the periodic part the angle subtended between the sun and the satellite at the planet, or a multiple of this angle. For a given mean motion of the satellite the angle varies more rapidly when the motion is retrograde than when it is direct, and the integrated perturbation will be correspondingly smaller. This may be stated algebraically. Let n, n' be the mean motions of the satellite and planet. The largest terms in the perturbative function contain the argument $(n - n')t$. On integration the factor $\frac{1}{n - n'}$ appears. This seems to give the reason why Hill's series expanded in powers of the parameter $\frac{n'}{n - n'}$ are more convergent than Delaunay's in powers of $\frac{n'}{n}$. The single objection that can be raised against Hill's parameter

* See the *Observatory* for 1913 June, p. 267.

is that it is numerically larger than Delaunay's. But this is only true for direct satellites, the opposite being the case for retrograde ones. For example, if $\frac{n'}{n} = +\frac{1}{3}$, $\frac{n'}{n-n'} = +\frac{1}{2}$; but if $\frac{n'}{n} = -\frac{1}{3}$, $\frac{n'}{n-n'} = -\frac{1}{4}$. On account, then, of the greater convergency of Hill's than of Delaunay's series, and on account of the smallness of $\frac{n'}{n-n'}$ for a negative as compared with an equal but positive value of $\frac{n'}{n}$, we see that at least the large perturbation known as the variation will be comparatively small for retrograde motion.*

Perhaps the following consideration, better than any other, will explain the great stability of retrograde satellites, especially when they are very remote from their primaries. Let two bodies with the same mean motion and the same mean longitude move round the sun without influence on each other. Suppose the one A to move in a circle, the other B to move in an ellipse. Introduce axes rotating with the mean motion of the two bodies. A will remain fixed relative to these axes, while B will describe an orbit relative to A, as shown in fig. 3. This figure is constructed for B moving in ellipses of eccentricity $\frac{1}{8}$, $\frac{2}{8}$, . . . $\frac{6}{8}$. The important point to notice is that this orbit of B relative to A is always described in the retrograde sense. In fact, each of the bodies, A, B, could be considered (relative to the rotating axes) a retrograde satellite of the other, although all the forces considered are due to solar attraction. The synodic period of the satellite would be the period of the planet round the sun. The same result follows when A does not move in a circle; two bodies, B_1, B_2, could each be considered a retrograde satellite of the other. This argument might seem of no use for our purpose, as, relative to axes fixed in direction, B does not move round A at all. But in the consideration of the solar perturbations of a satellite it is the angle subtended at the planet by the sun and the satellite which is of importance, and therefore rotating axes should be introduced.

It has been supposed that the bodies A, B do not act on each other; but no satellite in the solar system is so far from its primary that the latter does not completely control the motion. The ratio of the solar to the planetary force on a satellite is, in the ordinary notation, $\frac{m'}{m}\left(\frac{r}{r'}\right)^2$. The average value of this ratio is about 2 for the moon and about 1 for J VIII. For axes moving with the planet, but fixed in direction, the ratio of the solar perturbation to the planetary force is $2\frac{m'}{m}\left(\frac{r}{r'}\right)^3$ at syzygies, where it has a maximum value. This may be taken as the solar force for rotating axes. In the case of the moon the ratio has

* *Cf.* II., where "variational curves" for direct and retrograde satellites are given.

a value of about $\frac{1}{100}$; for J VIII it varies from about $\frac{1}{7}$ (at apojove) to $\frac{1}{60}$ (at perijove). At quadratures the perturbation is a minimum with half the value for syzygies. We thus see that in the case of J VIII the action of the sun is quite considerable; and, relative to the rotating axes, it tends to produce retrograde motion. It might be possible for a direct satellite to exist as remote from the planet as J VIII, but it would suffer tremendous perturbations. A little further away from Jupiter than J VIII the solar force would prohibit direct motion, but the solar and Jovian

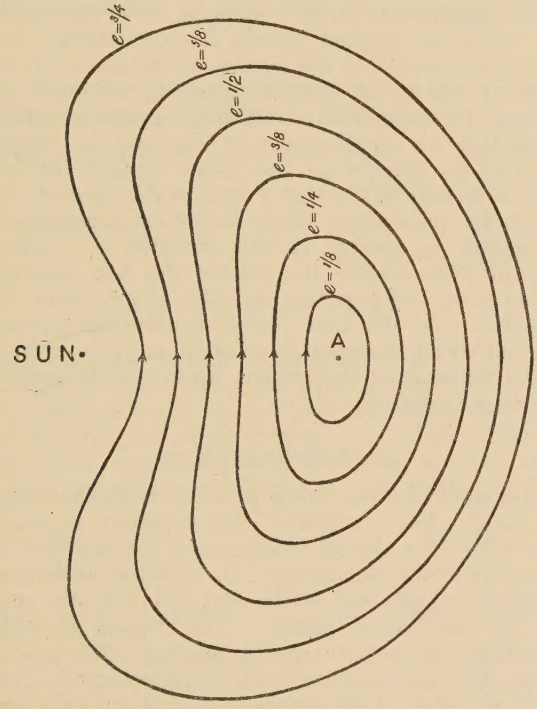


FIG. 3.—Elliptic Orbit referred to Rotating Axes.

forces would go hand in hand to maintain a retrograde satellite. It is quite conceivable that Jupiter might have a retrograde satellite at a much greater distance even than J VIII, but no such direct satellite could permanently exist.

Since a remote retrograde satellite is less perturbed than a direct one, it is worth while considering whether Jacobi's integral could prove a direct satellite with the same mean motion as J VIII to be permanently attached to Jupiter. The value of C for such a satellite * comes out so near the critical value as to leave it doubtful whether Jacobi's integral could prove stability. As the

* See p. 78.

perturbations of J VIII are smaller than for this hypothetical satellite, we seem to have good reasons for supposing J VIII to be a permanent member of the Jovian system.

With regard to general conclusions, it seems probable that amongst remote satellites retrograde motion is more stable than direct. This would give a reason for the existence of Phœbe and J VIII. Cosmogonists in general seem to have preferred to explain the solar system as it now is by the gradual action of forces through long intervals of time, the present system having been slowly evolved by a continuous process extending through ages. The writer would prefer to explain our system as the stable form to which an original nebula has condensed. Instead of explaining peculiarities in the system as produced by forces the magnitude of which can only be guessed, he would prefer to explain them by principles of stability—principles which must have controlled the development of the system. Suppose, for example, that at some stage the planet Jupiter had condensed within the solar nebula and that a considerable quantity of matter revolved in its proximity. There would be a general tendency for this matter to revolve in the direction of Jupiter's rotation, but the more remote the matter the less marked would this tendency be. The matter near the planet would form the inner satellites. Of the more remote matter that portion with a direct motion would be so perturbed as either to be precipitated on Jupiter or expelled from the system, while the retrograde portion would produce a satellite.

II. *Variational Curves.*

It is desirable that the orbits of some retrograde satellites be actually traced. For simplicity it is necessary to consider the planet to move in a circular orbit round the sun and the three bodies to move in the same plane. In Darwin's work the mass of Jove was fixed at $\frac{1}{10}$th that of the sun, and then the orbits were obtained by mechanical means. The results therefore show rigorous orbits, but they are special, as they refer to a planet of fixed mass. Hill, on the other hand, by neglecting the "parallactic terms" and the mass of the planet in comparison with the mass of the sun, obtained results which, though not completely rigorous for remote satellites, are applicable to any relative masses of the sun and planet. The only alteration necessary to make the results valid for all planets is a change of the scale on which the orbits are traced. Hill's method is algebraic, and we do not scruple to use it, as it leads to results with great rapidity.

As already stated, Hill expanded the functions in ascending powers of $m=\frac{n'}{n-n'}$, but, finding that the series were still more convergent if $m'=\frac{3n'}{3n-4n'}$ were used, he expressed some of his series in terms of this parameter. A glance at the series in m'

shows that the convergence would be greater if we were to write $-m'$ for $+m'$. Hence if the terms obtained are sufficient to trace the orbit for $\frac{3n'}{3n-4n'}=k$, they will be sufficient for $\frac{3n'}{3n-4n'}=-k$. Now, Hill has applied the series for $\frac{n}{n'}=4$, *i.e.* for $k=\frac{3}{8}$; therefore they can be used for $\frac{3n'}{3n-4n'}=-\frac{3}{8}$, *i.e.* $\frac{n}{n'}=-\frac{4}{3}$. In fact, there can be no doubt that for purposes of plotting curves the series are sufficiently convergent to deal with all retrograde satellites to $\frac{n}{n'}=-1$.

Let r, θ be the polar co-ordinates of the satellite referred to rotating axes centred at the planet. Hill obtained *

$$r\cos(\theta-\overline{n-n'}t)=a_0\{1+[-m'^2-\tfrac{1}{2}m'^3+\tfrac{2}{9}m'^4\ \ldots]\cos 2(n-n')t + \tfrac{25}{256}m'^4\cos 4(n-n')t\ \ldots\}$$

$$r\sin(\theta-\overline{n-n'}t)=a_0\{+[\tfrac{11}{8}m'^2+\tfrac{5}{4}m'^3+\tfrac{5}{72}m'^4\ \ldots]\sin 2(n-n')t + \tfrac{25}{256}m'^4\sin 4(n-n')t\ \ldots\}$$

$$a_0=\frac{1}{n^{\frac{2}{3}}}\{1-\tfrac{1}{6}m'^2+\tfrac{4}{9}m'^3-\tfrac{163}{768}m'^4\ \ldots\}$$

the series being given to include terms up to m'^9.

By means of these series we have obtained the variational curves for retrograde motion, and give for comparison the results calculated by Hill for direct motion. It will be seen that the curves are much more circular and the series much more convergent when the motion is retrograde. τ is written for $(n-n')t$.

	Retrograde.	*Direct.*
(1)	$n=-5n'$	$n=+5n'$
$\frac{x}{a_0}=$	·973 871 cos τ	·894 792 cos τ
	+ 3 299 cos 3τ	+ 21 970 cos 3τ
	+ 37 cos 5τ	+ 1 070 cos 5τ
	+ 1 cos 7τ	+ 68 cos 7τ
$\frac{y}{a_0}=$	1·026 129 sin τ	1·105 208 sin τ
	+ 3 304 sin 3τ	+ 21 805 sin 3τ
	+ 37 sin 5τ	+ 1 057 sin 5τ
	+ 1 sin 7τ	+ 67 sin 7τ
	$\log a_0=9{\cdot}5313951$	$\log a_0=9{\cdot}5318013$
$\frac{y \text{ for } \tau=\pi/2}{x \text{ for } \tau=0}=$	·347706 / ·332186	·36897 / ·31232
	$=1{\cdot}04672$	$=1{\cdot}18138$

* *Mathematical Works*, vol. i. pp. 371 *et seq.*

(2) $n = -4n'$

$$\frac{x}{a_0} = \cdot 964\ 074 \cos \tau + 4\ 323 \cos 3\tau + 66 \cos 5\tau + 1 \cos 7\tau$$

$$\frac{y}{a_0} = 1 \cdot 035\ 926 \sin \tau + 4\ 333 \sin 3\tau + 66 \sin 5\tau + 1 \sin 7\tau$$

$\log a_0 = 9 \cdot 5946865$

$$\frac{y \text{ for } \tau = \pi/2}{x \text{ for } \tau = 0} = \frac{\cdot 405716}{\cdot 380865} = 1 \cdot 065250$$

$n = +4n'$

$$\frac{x}{a_0} = \cdot 791\ 746 \cos \tau + 46\ 474 \cos 3\tau + 4\ 491 \cos 5\tau + 551 \cos 7\tau + 76 \cos 9\tau + 11 \cos 11\tau$$

$$\frac{y}{a_0} = 1 \cdot 208\ 253 \sin \tau + 45\ 566 \sin 3\tau + 4\ 385 \sin 5\tau + 540 \sin 7\tau + 75 \sin 9\tau + 11 \sin 11\tau$$

$\log a_0 = 9 \cdot 5955815$

$$\frac{y \text{ for } \tau = \pi/2}{x \text{ for } \tau = 0} = \frac{\cdot 45973}{\cdot 33235} = 1 \cdot 38329$$

(3) $n = -2 \cdot 78265n'$

$$\frac{x}{a_0} = \cdot 942\ 561 \cos \tau + 6\ 280 \cos 3\tau + 156 \cos 5\tau + 6 \cos 7\tau$$

$$\frac{y}{a_0} = 1 \cdot 057\ 439 \sin \tau + 6\ 300 \sin 3\tau + 156 \sin 5\tau + 6 \sin 7\tau$$

$\log a_0 = 9 \cdot 6962845$

$$\frac{y \text{ for } \tau = \pi/2}{x \text{ for } \tau = 0} = \frac{\cdot 522404}{\cdot 471576} = 1 \cdot 107782$$

$n = +2 \cdot 78265n'$

Mechanical quadratures

$$\frac{y \text{ for } \tau = \pi/2}{x \text{ for } \tau = 0} = \frac{\cdot 78190}{\cdot 27180} = 3 \cdot 87676$$

The last given direct orbit is the most remote that has been actually traced. Poincaré has demonstrated * that the next orbit of the series has a node at quadratures, but no such orbit has so far been mechanically computed. We give the orbit calculated for $n = -n'$.

* *Méthodes Nouvelles de la Mécanique Céleste*, i. p. 109.

(4) *Retrograde.*

$$\frac{x}{a_0} = \cdot85035\cos\tau + 1112\cos 3\tau + 115\cos 5\tau + 8\cos 7\tau$$

$$\frac{y}{a_0} = 1\cdot14965\sin\tau + 1082\sin 3\tau + 115\sin 5\tau + 9\sin 7\tau$$

$$\log a_0 = 9\cdot96629$$

$$\frac{y \text{ for } \tau=\pi/2}{x \text{ for } \tau=0} = \frac{1\cdot05476}{\cdot79827} = 1\cdot32130.$$

These series have been used to trace the curves given in fig. 4. The last orbit for which $n = -n'$ belongs to a satellite with the same mean motion as the primary. Hill's method can hardly be extended to include satellites with a smaller mean motion. However, the limiting solution for $n=0$ can be obtained. In this case the dimensions of the orbit are infinite, so that the planetary force is zero. Hill's equations of motion then are

$$\left.\begin{aligned} \frac{d^2x}{d\tau^2} + 2\frac{dy}{d\tau} - 3x &= 0 \\ \frac{d^2y}{d\tau^2} - 2\frac{dx}{d\tau} &= 0 \end{aligned}\right\}, \quad \tau = -n't.$$

The particular solution of these equations with which we are concerned is

$$\frac{x}{a_0} = \cos\tau, \qquad \frac{y}{a_0} = 2\sin\tau.$$

a_0 infinite

$$\frac{y \text{ for } \tau=\pi/2}{x \text{ for } \tau=0} = 2.$$

It should be noted that Hill's differential equations are not rigorous, and that this is really an approximate solution of the problem solved in fig. 3. The difference between the results indicates for remote satellites the importance of the terms neglected by Hill.

Attention may here be drawn to the following figures which illustrate the effect of solar perturbations on the size of an orbit for given mean motions:—

	Log a_0.		
n/n'.	Direct.	Retrograde.	Log a_0 for an Undisturbed Satellite.
± 5	9·53 180	9·53 140	9·53 402
± 4	9·59 558	9·59 469	9·59 863
$\pm 2\cdot78265$	...	9·69 628	9·70 369
± 1	...	9·96 629	·00 000

Diagram of Variational Curves.—The numbers in brackets (1), (3), (4) refer to the orbits (1), (3), (4) above, so that the mean

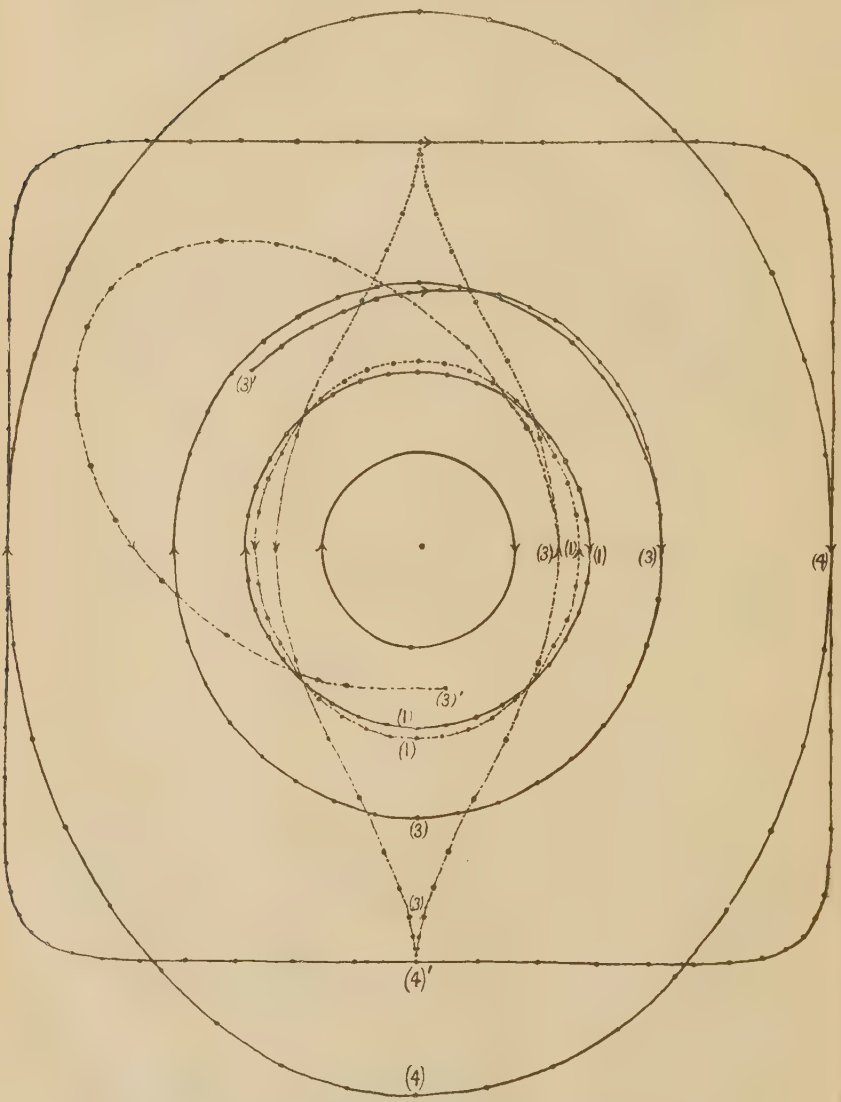


FIG. 4.—Variational Curves for Direct and Retrograde Satellites.
Continuous line: Retrograde. Dotted line: Direct.

motions of the satellites in terms of the mean motion of the primary are ± 5, $\pm 2{\cdot}78265$, -1. The orbits $(3)'$, $(4)'$ refer to

axes fixed in direction. The small inner orbit is that of Darwin's retrograde satellite. The "square orbit" is the variational curve for a retrograde satellite with the same mean motion as the primary, referred to axes fixed in direction.

The unit of length is Hill's theoretical unit (see III.).

For Jupiter, Hill's unit = ·512 astronomical units.
For Saturn, ,, ·628 ,,
For Earth, ,, ·01455 ,,

∴ For Jupiter, distance of sun from planet = 10·159 Hill's units.
For Saturn, ,, ,, 15·187 ,,
For Earth, ,, ,, 58·701 ,,

These latter numbers show that for Jupiter and Saturn the omission of the parallactic terms will considerably vitiate the more remote orbits.

III. *Numerical Data connected with* I.

A. *Units used by different authors in connection with Jacobi's Integral.*

A.U. = astronomical unit of length.
M.S.D. = mean solar day.

Numbers in brackets are logarithms.

$$\frac{\text{Mass of sun}}{\text{Mass of Jupiter}} = 1047{\cdot}35 \qquad [3{\cdot}020\ 092].$$

$$\frac{\text{Mass of sun}}{\text{Mass of Saturn}} = 3501{\cdot}6 \qquad [3{\cdot}544\ 266].$$

(1) *Hill.*

Gravitation constant = 1.
Mass of planet = 1.
Mean motion of planet = 1.

For *Jupiter*:
Unit of length = ·512155 A.U. [9·709 402 − 10].
Unit of time = 689·552 M.S.D. [2·838 567].

For *Saturn*:
Unit of length = ·628105 A.U. [9·798 032 − 10].
Unit of time = 1712·38 M.S.D. [3·233 601].

(2) *Darwin.*

Gravitation constant = 1.
Mass of planet = 1.
Distance between sun and planet = 1.

For *Jupiter*:
Mean motion of planet = 32·3782 [1·510 253].
Unit of time = 22326·5 M.S.D. [4·348 821].

For *Saturn*:
Mean motion of planet = 59·1828 [1·772 195].
Unit of time = 101343·5 M.S.D. [5·005 796].

(3) *Moulton.*

Gravitation constant = 1.
Sum of masses of sun and planet = 1.
Distance from sun to planet = 1.
∴ Mean motion of planet = 1.

For *Jupiter*:
Unit of time = 689·552 M.S.D. [2·838 567].

For *Saturn*:
Unit of time = 1712·38 M.S.D [3·233 601].

(4) *Ordinary Units.*

Gravitation constant = 1.
Unit of time = 1 M.S.D.
Unit of length = 1 A.U.
∴ Mass of sun $= k^2$, where $\log k =$ [8·235 581 − 10].

For *Jupiter*:
Mean motion of planet = ·00145022 [7·161 433 − 10].

For *Saturn*:
Mean motion of planet = ·00058398 [6·766 399 − 10].

Jacobi's integral in the various notations runs:—

(1) *Hill*:*

$$\frac{\mu}{\sqrt{x^2+y^2+z^2}}+\frac{n'^2a'^3}{\sqrt{(a'-x)^2+y^2+z^2}}=\mathrm{C}+\tfrac{3}{2}n'^2a'^2-\tfrac{1}{2}[(a'-x)^2+y^2]+\tfrac{1}{2}\mathrm{V}^2$$
$$(\mu=1,\ n'=1).$$

(2) *Darwin*: †

$$\mathrm{V}^2=2\Omega-\mathrm{C} \text{ where } 2\Omega=\left(r^2+\frac{2}{r}\right)+\nu\left(\rho^2+\frac{2}{\rho}\right)$$
$$\left(\nu=\frac{\text{mass of sun}}{\text{mass of planet}}\right).$$

(3) *Moulton*: ‡

$$(1-\mu)\left(r'^2+\frac{2}{r'}\right)+\mu\left(r^2+\frac{2}{r}\right)=\mathrm{V}^2+\mathrm{C}'=\mathrm{V}^2+\mathrm{C}+\mu(1-\mu)$$
$$\left(\frac{1-\mu}{\mu}=\frac{\text{mass of sun}}{\text{mass of planet}}\right).$$

Hence we have

$$\mathrm{C(Darwin)}=\mathrm{C}+\mu(1-\mu)(\text{Moulton})=\mathrm{C}'\ (\text{Moulton})$$
$$=2\mathrm{C}+3a'^2\ (\text{Hill}).$$

The dimensions of C are

$$\frac{[\mathrm{M}]}{[\mathrm{L}]} \text{ or } \frac{[\mathrm{L}]^2}{[\mathrm{T}]^2}.$$

* *Mathematical Works*, vol. i. p. 297.
† *Scientific Papers*, vol. iv. p. 4.
‡ *Celestial Mechanics*, pp. 186 *et seq.*, p. 194.

Hence, numerically,

For *Jupiter*:

$$\begin{aligned} C(\text{Darwin}) &= 1048{\cdot}35\{C + {\cdot}00095297\} \text{ (Moulton)} \\ &\quad [3{\cdot}020506] \\ &= 10{\cdot}15864\{2C + 309{\cdot}5937\} \text{ (Hill)} \\ &\quad [1{\cdot}006835]. \end{aligned}$$

For *Saturn*:

$$\begin{aligned} C(\text{Darwin}) &= 3502{\cdot}6\{C + {\cdot}00028542\} \text{ (Moulton)} \\ &\quad [3{\cdot}544390] \\ &= 15{\cdot}18670\{2C + 691{\cdot}9080\} \text{ (Hill)} \\ &\quad [1{\cdot}181463]. \end{aligned}$$

It is to be noted that the form of Jacobi's integral used by Darwin and Moulton is rigorous, but that used by Hill is only approximate.

B. *Critical Values of the Constants* (Moulton's notation).

See fig. 2.

	Jupiter.	Saturn.
Distance of P_1 from planet .	$-{\cdot}066\ 681$	$-{\cdot}044\ 953$
,, P_2 ,, .	$+{\cdot}069\ 784$	$+{\cdot}046\ 343$
Constant for curve C_1 . .	$3{\cdot}039\ 713$	$3{\cdot}018\ 097$
,, ,, C_2 . .	$3{\cdot}038\ 442$	$3{\cdot}017\ 716$

C. *Cases of Phœbe and J VIII* (Moulton's notation).

The constant is to be calculated from

$$C + \mu(1-\mu) = (1-\mu)\left(\rho^2 + \frac{2}{\rho}\right) + \mu\left(r^2 + \frac{2}{r}\right) - V^2.$$

As Saturn and Jupiter do not move in circles round the sun, Jacobi's integral cannot be rigorously applied. Using mean values for the quantities, we get

$$C + \mu(1-\mu) = (1-\mu)(1+2) + \mu\left(a^2 + \frac{2}{a}\right) - a^2(1-n)^2$$

where n = mean motion of satellite
a = ,, distance ,,
$1-\mu$ = mass of sun in terms of sum of masses of sun and planet
μ = ,, planet ,, ,, ,,

Phœbe:

$n = -0^\circ{\cdot}65394$ per M.S.D.

$a = \frac{1872''{\cdot}4}{206265''}$ in terms of mean distance between sun and Saturn.

$\log a = 7{\cdot}95797-10$ (data from *Conn. des Temps*).

$$\begin{aligned}
\therefore\ C+\mu(1-\mu) = \quad & 3 && = 3{\cdot}000\ 0000\\
& -3\mu && - \cdot000\ 8565\\
& +\mu a^2 && + \cdot000\ 0000\\
& +\frac{2\mu}{a^2} && + \cdot062\ 9022\\
& -a^2(1-n)^2 && - \cdot034\ 7794\ *\\
& && = 3{\cdot}027\ 2663\\
& \mu(1-\mu) && = \cdot000\ 2854\\
\therefore\ & C(\text{Moulton}) && = 3{\cdot}026\ 9809
\end{aligned}$$

J VIII:

$a = \cdot156707$ A.U.
$n = -1748''{\cdot}19$ per M.S.D.

$$\begin{aligned}
C+\mu(1-\mu) = \quad & 3 && = 3{\cdot}000\ 0000\\
& -3\mu && - \cdot002\ 8616\\
& +\mu a^2 && + \cdot000\ 0009\\
& +\frac{2\mu}{a} && + \cdot063\ 3392\\
& -a^2(1-n)^2 && - \cdot042\ 4970\\
& && 3{\cdot}017\ 9815\\
& \mu(1-\mu) && = \cdot000\ 9530\\
\therefore\ & C(\text{Moulton}) && = 3{\cdot}017\ 0285
\end{aligned}$$

A hypothetical direct satellite with same mean motion as J VIII:

$$\begin{aligned}
\text{For } a^2(1-n)^2 \text{ in above use } a^2(1+n)^2 = \quad & \cdot021\ 2893.\\
\therefore\ C(\text{Moulton}) = \quad & 3{\cdot}038\ 2362.
\end{aligned}$$

D. *The constant for J VIII according to various methods.*

If we use Hill's † series for C and take

$$\frac{n_J}{n_{VIII}-n_J} = -\cdot146107, \qquad \frac{n_{VIII}}{n_J} = -5{\cdot}84428,$$

we get

$$\begin{aligned}
2C(\text{Hill}) \quad &= 2{\cdot}2421.\\
\therefore\ C(\text{Moulton}) \quad &= 3{\cdot}02173.
\end{aligned}$$

The actual co-ordinates and velocities of J VIII at different times being available, the constant has been determined at various times. It has been necessary to introduce a mean sun revolving round Jupiter, and the fact that this differs from the real sun explains the differences between the constants at different times.

r is the distance from satellite to planet in terms of mean

* This is the term apparently neglected by Moulton.
† *Mathematical Works*, vol. i. p. 320.

distance of Jupiter from the sun, Δ the distance from satellite to mean sun. The principal results are

	1908 Feb. 29.	1909 Jan. 14.	1909 Nov. 30.	1910 Oct. 16.
$(1-\mu)\left(\Delta^2+\frac{2}{\Delta}\right)$	3·001 0124	2·997 2359	2·999 7083	2·997 6400
$\mu(r^2+2/r)$	·052 0588	·076 8721	·045 9678	·103 1102
$-V^2$	−·032 0369	−·053 4042	−·024 6187	−·079 9288
$C+\mu(1-\mu)$ (Moulton)	3·021 0343	3·020 7038	3·021 0574	3·020 8214
∴ C(Moulton)	3·020 0813	3·019 7508	3·020 1044	3·019 8684
∴ 2C(Hill)	2·1707	2·1366	2·1731	2·1488

This gives a mean value of C(Moulton) = 3·01995
2 C(Hill) = 2·1573

This is somewhat smaller than the value derived by Hill's series. In the series no allowance is made for inclination of the plane of the satellite's motion, while V has been calculated as the actual velocity relative to rotating axes, the third dimension having been introduced. We may therefore take the above means as the best values of the constant of Jacobi's integral for J VIII.

IV. *The Accuracy of Hill's Form of Jacobi's Integral.*

Instead of the rigorous form of Jacobi's integral, Hill uses an approximation. It is interesting to discuss analytically the error introduced. If we put V = 0, and introduce Moulton's units in which the sum of the masses of sun and planet is taken as unity, Hill's equation reads,

$$C=\frac{2\mu}{r}+\left(\frac{2}{\rho}+\rho^2\right)$$

instead of

$$C=\mu\left(\frac{2}{r}+r^2\right)+(1-\mu)\left(\frac{2}{\rho}+\rho^2\right).$$

Let ρ_1, ρ_2 be the distances of the double points P_1, P_2 (see fig. 2, p. 66) from the planet. We have the exact equations

$$\left.\begin{aligned} C_1&=\mu\left(\frac{2}{\rho_1}+\rho_1^2\right)+(1-\mu)\left\{\frac{2}{(1-\rho_1)}+(1-\rho_1)^2\right\}\\ 0&=\mu\left(\frac{-2}{\rho_1^2}+2\rho_1\right)+(1-\mu)\left\{\frac{2}{(1-\rho_1)^2}-2(1-\rho_1)\right\}\end{aligned}\right\}. \quad (1)$$

and

$$\left.\begin{aligned} C_2&=\mu\left(\frac{2}{\rho_2}+\rho_2^2\right)+(1-\mu)\left\{\frac{2}{(1+\rho_2)}+(1+\rho_2)^2\right\}\\ 0&=\mu\left(\frac{-2}{\rho_2^2}+2\rho_2\right)+(1-\mu)\left\{\frac{-2}{(1+\rho_2)^2}+2(1+\rho_2)\right\}\end{aligned}\right\}. \quad (2)$$

These determine the values of ρ_1, ρ_2 and the corresponding values of C.

(1) If we expand the second equation in ascending powers of ρ_1, and multiply by $\rho_1^2/2$ we get

$$(1-\mu)\{3\rho_1^3+3\rho_1^4+4\rho_1^5+5\rho_1^6+\ \ldots\ \}=\mu(1-\rho_1^3).$$

Write $\mu=3\lambda^3$ and assume $\rho_1=\lambda+\beta\lambda^2+\gamma\lambda^3+\delta\lambda^4\ \ldots$

Then

$$\begin{aligned}(1-3\lambda^3)\{&3[\lambda^3+3\beta\lambda^4+(3\gamma+3\beta^2)\lambda^5+(3\delta+6\beta\gamma+\beta^3)\lambda^6]\\&+3[\lambda^4+4\beta\lambda^5+(4\gamma+6\beta^2)\lambda^6]\\&+[4\lambda^5+(20\beta+5)\lambda^6]\ \ldots\ \}=3\lambda^3(1-\lambda^3).\end{aligned}$$

Equating coefficients we get

$$\left.\begin{aligned}9\beta+3&=0\\3\gamma+3\beta^2+4\beta+\tfrac{4}{3}&=0\\-3+3\delta+6\beta\gamma+\beta^3+4\gamma+6\beta^2+\tfrac{20}{3}\beta+\tfrac{5}{3}&=-1\end{aligned}\right\}$$

$$\therefore\quad \beta=-\tfrac{1}{3},\quad \gamma=-\tfrac{1}{9},\quad \delta=+\tfrac{58}{81}.$$

$$\therefore\quad \rho_1=\lambda-\tfrac{1}{3}\lambda^2-\tfrac{1}{9}\lambda^3+\tfrac{58}{81}\lambda^4+\ \ldots$$

The corresponding value of the constant is given (on elimination of μ/ρ_1) by

$$\begin{aligned}C_1&=3\mu\rho_1^2+(1-\mu)\left\{\frac{2}{(1-\rho_1)^2}+1-4\rho_1+3\rho_1^2\right\}\\&=9\lambda^3\rho_1^2+(1-3\lambda^3)\{3+9\rho_1^2+8\rho_1^3+10\rho_1^4+12\rho_1^5\ \ldots\ \}\\&=9\lambda^5+(1-3\lambda^3)\{3+9[\lambda^2-\tfrac{2}{3}\lambda^3-\tfrac{1}{9}\lambda^4+\tfrac{122}{81}\lambda^5]\\&\qquad+8[\lambda^3-\lambda^4]+10[\lambda^4-\tfrac{4}{3}\lambda^5]+12\lambda^5\}\ \ldots\\&=3+9\lambda^2-7\lambda^3+\lambda^4-\tfrac{52}{9}\lambda^5\ \ldots\end{aligned}$$

(2) ρ_2 is given by

$$(1-\mu)\{3\rho_2^3-3\rho_2^4+4\rho_2^5-5\rho_2^6+\ \ldots\ \}=\mu(1-\rho_2^3).$$

As before, write $\mu=3\lambda^3$ and assume $\rho_2=\lambda+\beta\lambda^2+\gamma\lambda^3+\delta\lambda^4$.

Then

$$\begin{aligned}(1-3\lambda^3)\{&3[\lambda^3+3\beta\lambda^4+(3\gamma+3\beta^2)\lambda^5+(3\delta+6\beta\gamma+\beta^3)\lambda^6]\\&-3[\lambda^4+4\beta\lambda^5+(4\gamma+6\beta^2)\lambda^6]\\&+4\lambda^5+(20\beta-5)\lambda^6\ \ldots\}=3\lambda^3(1-\lambda^3).\end{aligned}$$

Equating coefficients

$$\left.\begin{aligned}9\beta-3&=0\\3\gamma+3\beta^2-4\beta+\tfrac{4}{3}&=0\\-3+3\delta+6\beta\gamma+\beta^3-4\gamma-6\beta^2+\tfrac{20}{3}\beta-\tfrac{5}{3}&=-1\end{aligned}\right\}$$

$$\therefore\quad \beta=+\tfrac{1}{3},\quad \gamma=-\tfrac{1}{9},\quad \delta=+\tfrac{50}{81},$$

$$\therefore\quad \rho_2=\lambda+\tfrac{1}{3}\lambda^2-\tfrac{1}{9}\lambda^3+\tfrac{50}{81}\lambda^4,$$

and

$$\begin{aligned}C_2&=3\mu\rho_2^2+(1-\mu)\left\{\frac{2}{(1+\rho_2)^2}+1+4\rho_2+3\rho_2^2\right\}\\&=9\lambda^3\rho_2^2+(1-3\lambda^3)\{3+9\rho_2^2-8\rho_2^3+10\rho_2^4-12\rho_2^5+\ \ldots\}\\&=9\lambda^5+(1-3\lambda^3)\{3+9[\lambda^2+\tfrac{2}{3}\lambda^3-\tfrac{1}{9}\lambda^4+\tfrac{24}{81}\lambda^5]\\&\qquad-8[\lambda^3+\lambda^4]+10[\lambda^4+\tfrac{4}{3}\lambda^5]-12\lambda^5\}\ \ldots\\&=3+9\lambda^2-11\lambda^3+\lambda^4-\tfrac{56}{9}\lambda^5.\end{aligned}$$

In discussing Hill's form of the integral we use accented letters.

$$\left.\begin{aligned} C_1' &= \frac{2\mu}{\rho_1'} + \frac{2}{(1-\rho_1')} + (1-\rho_1')^2 \\ 0 &= -\frac{2\mu}{\rho_1'^2} + \frac{2}{(1-\rho_1')^2} - 2(1-\rho_1') \end{aligned}\right\} \quad . \quad . \quad (1)'$$

$$\left.\begin{aligned} C_2' &= \frac{2\mu}{\rho_2'} + \frac{2}{(1+\rho_2')} + (1+\rho_2')^2 \\ 0 &= -\frac{2\mu}{\rho_2'^2} - \frac{2}{(1+\rho_2')^2} + 2(1+\rho_2') \end{aligned}\right\} \quad . \quad . \quad (2)'$$

(1)′ ρ_1' is given by

$$3\lambda^3 = 3\rho_1'^3 + 3\rho_1'^4 + 4\rho_1'^5 + 5\rho_1'^6 + \ldots$$

If we assume $\rho_1' = \lambda + \beta\lambda^2 + \gamma\lambda^3 + \delta\lambda^4$ we get

$$\begin{aligned} 3\lambda^3 = {} & 3[\lambda^3 + 3\beta\lambda^4 + (3\gamma + 3\beta^2)\lambda^5 + (3\delta + 6\beta\gamma + \beta^3)\lambda^6] \\ & + 3[\lambda^4 + 4\beta\lambda^5 + (4\gamma + 6\beta^2)\lambda^6] + 4[\lambda^5 + 5\beta\lambda^6] + 5[\lambda^6] \ldots \end{aligned}$$

$$\therefore \quad \left.\begin{aligned} 3\beta + 1 &= 0 \\ 3\gamma + 3\beta^2 + 4\beta + \tfrac{4}{3} &= 0 \\ 3\delta + 6\beta\gamma + \beta^3 + 4\gamma + 6\beta^2 + \tfrac{20}{3}\beta + \tfrac{5}{3} &= 0 \end{aligned}\right\}$$

$$\therefore \quad \beta = -\tfrac{1}{3}, \quad \gamma = -\tfrac{1}{9}, \quad \delta = +\tfrac{4}{81},$$

$$i.e. \quad \rho_1' = \lambda - \tfrac{1}{3}\lambda^2 - \tfrac{1}{9}\lambda^3 + \tfrac{4}{81}\lambda^4,$$

and

$$\begin{aligned} C_1' &= \frac{2}{(1-\rho_1')^2} + 1 - 4\rho_1' + 3\rho_1'^2 \\ &= 3 + 9\rho_1'^2 + 8\rho_1'^3 + 10\rho_1'^4 + 12\rho_1'^5 + \ldots \\ &= 3 + 9[\lambda^2 + \tfrac{2}{3}\lambda^3 - \tfrac{1}{9}\lambda^4 + \tfrac{14}{81}\lambda^5] + 8[\lambda^3 - \lambda^4] + 10[\lambda^4 - \tfrac{4}{9}\lambda^5] + 12\lambda^5 \ldots \\ &= 3 + 9\lambda^2 + 2\lambda^3 + \lambda^4 + \tfrac{2}{9}\lambda^5. \end{aligned}$$

(2)′ Similarly we obtain

$$\rho_2' = \lambda + \tfrac{1}{3}\lambda^2 - \tfrac{1}{9}\lambda^3 - \tfrac{4}{81}\lambda^4$$

$$C_2' = 3 + 9\lambda^2 - 2\lambda^3 + \lambda^4 - \tfrac{2}{9}\lambda^5.$$

Summing up, if $3\lambda^3 = \dfrac{\text{mass of planet}}{\text{sum of masses of sun and planet}}$, and if the distances be measured in terms of the distance between sun and planet, the radii vectores from the planet towards the critical points P_1, P_2 (fig. 2) are respectively

$$\rho_1 = \lambda - \tfrac{1}{3}\lambda^2 - \tfrac{1}{9}\lambda^3 + \tfrac{58}{81}\lambda^4 + \ldots$$

and

$$\rho_2 = \lambda + \tfrac{1}{3}\lambda^2 - \tfrac{1}{9}\lambda^3 + \tfrac{50}{81}\lambda^4 + \ldots$$

and the corresponding values of the constant are

$$C_1 = 3 + 9\lambda^2 - 7\lambda^3 + \lambda^4 - \tfrac{52}{9}\lambda^5$$

and

$$C_2 = 3 + 9\lambda^2 - 11\lambda^3 + \lambda^4 - \tfrac{56}{9}\lambda^5.$$

Hill's approximate equation leads to

$$\rho_1' = \rho_1 - \tfrac{2}{3}\lambda^4, \qquad \rho_2' = \rho_2 - \tfrac{2}{3}\lambda^4,$$
$$C_1' = C_1 + 9\lambda^3 + 6\lambda^5, \quad C_2' = C_2 + 9\lambda^3 + 6\lambda^5,$$

and therefore

$$\rho_1' - \rho_1 = \rho_2' - \rho_2\,; \; C_1' - C_1 = C_2' - C_2,$$

with the accuracy of our approximation.

The following shows the importance of these quantities for members of the solar system :—

	Jupiter.		Saturn.	
λ.	+·068	253	+·045	656
$\rho_1 - \rho_1' = \rho_2 - \rho_2' = \frac{2}{3}\lambda^4$	+·000	015	+·000	003
$C_1 - C_1' = C_2 - C_2' = 9\lambda^3 + 6\lambda^5$	+·002	870	+·000	858
$\rho_1 - \rho_2 = \rho_1' - \rho_2 = -\frac{2}{3}\lambda^2 + \frac{8}{81}\lambda^4$	−·003	104	−·001	389
$C_1 - C_2 = C_1' - C_2' = 4\lambda^3 + \frac{4}{3}\lambda^5$	+·001	272	+·000	381

On a Simple Method of detecting Discontinuities in a Series of Recorded Observations, with an application to Sun-spots, suggesting that they are caused by a Meteor Swarm due to Successive Encounters of the Leonids with Saturn, which has been more than once considerably perturbed by the Leonid Swarm. (*Papers of the I.U.S.R. Computing Bureau, No. XIII.*) By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In Nos. X. and XII. of this series of papers (*M.N.*, lxxiii. p. 714, and lxxiv. p. 16) I have considered the result of analysing Wolf's Sun-spot numbers in a Fourier sequence, arriving finally at the conclusion that none of the periodicities disclosed were continuous throughout the sequence. At the end of the second paper I had begun to suspect that there were abrupt changes in the series, and the suspicion arose in this way. Having identified four main periodicities, the coefficients of which seemed nevertheless to vary, I tried the effect of removing three of them by the best supposition I could make as to the continuous variation of the coefficients of these three, leaving the fourth untouched. The result was striking. The three removed were of periods $14\frac{2}{3}$, $11\frac{2}{5}$, and $10\frac{1}{4}$ years, leaving the shortest period of $8\frac{1}{3}$ years. It then appeared as a series of about five well-marked oscillations at the beginning of the series. There is scarcely any need to give the diagram, striking though it is: for the explanation of it which will appear in the sequel will enable anyone to imagine it.

2. This suggested at first that the first forty years were quite

different from the rest; that perhaps Wolf's numbers were in these early years not good enough for use; and that it would be well in any case to try the effect of excluding them. Various experiments were tried on the diminished series, without advancing matters much beyond the suspicion of another discontinuity about 1830. I felt that it was unsatisfactory to assume such breaks in the series without some criterion for exhibiting them clearly; and the following simple expedient occurred to me, which has proved of value in this case and may be useful elsewhere. Hence I give the steps in some detail, though the principle is easily apprehended.

Method for detecting Discontinuities.

3. [The method will be gathered as readily from an example as from a general statement, and I shall describe the actual example.] Take a single *arbitrary* period, in this case 12 years. Sun-spots are suspected of periods *near* this, whether 8 years or 14 years, etc., does not matter for our present purpose. Take then the first twelve years and form S_1 and C_1, the coefficients of the first harmonic.

Now instead of year 1, substitute year 13: and form S_2, C_2. Notice that we do not begin with year 2, for this would change the phase unnecessarily. The series is 13, 2, 3, 4, . . . 12.

Now substitute year 14 for 2, so that the series becomes 13, 14, 3, 4, . . . 12: and form S_3, C_3.

Continue this process until all the series of terms is used up. If N be the whole number of terms we shall clearly have N—11 values of S and C.

4. This sounds like a considerable piece of work, but it is easily negotiated as follows:—

TABLE I.

Wolf's Numbers.

Year.	o	o	o	o	o	o	o	o	o	o	o	o
1749	81	83	48	48	31	12	10	10	32	48	54	63
1761	86	61	45	36	21	11	38	70	106	101	82	66
1773	35	31	7	20	92	154	126	85	68	38	23	10
1785	24	83	132	131	118	90	67	60	47	41	21	16
etc.				etc.					etc.			

TABLE II.

Differences of Table I.

1749	81	83	48	48	31	12	10	10	32	48	54	63
1761	+ 5	−22	− 3	− 12	−10	− 1	+28	+60	+74	+53	+28	+ 3
1773	−51	−30	− 38	− 16	+71	+143	+88	+15	−38	−63	−59	−56
1785	−11	+52	+125	+111	+26	− 64	−59	−25	−21	+ 3	− 2	+ 6
etc.				etc.					etc.			

TABLE III.

Products for Sines.

1749	0	+42	+ 41	+ 48	+27	+ 6	0	− 5	−28	−48	−47	−32
1761	0	−11	− 3	− 12	− 9	0	0	−30	−65	−53	−24	− 2
1773	0	−15	− 33	− 16	+62	+72	0	− 8	+33	+63	+51	+28
1785	0	+26	+109	+111	+22	−32	0	+12	+18	− 3	+ 2	− 3

TABLE IV.

Products for Cosines.

1749	+81	+72	+24	0	−16	− 10	−10	− 9	−16	0	+27	+55
1761	+ 5	−19	− 2	0	+ 5	+ 1	−28	−52	−37	0	+14	+ 3
1773	−51	−26	−19	0	−36	−124	−88	−13	+19	0	−30	−48
1785	−11	+45	+62	0	−13	+ 56	+59	+22	+10	0	− 1	+ 5

Write down, as in Table I., Wolf's numbers in sets of twelve, with a series of zeros at the top. Subtract each row of Table I. from the row below it to get Table II. Multiply the columns of Table II. by the appropriate sine and cosine factors for the first harmonic, viz.,

for sine	0	$\frac{1}{2}$	f	1	f	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-f$	−1	$-f$	$-\frac{1}{2}$
for cosine	1	f	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-f$	−1	$-f$	$-\frac{1}{2}$	0	$\frac{1}{2}$	f

where $f = \sqrt{3}/2 = \cdot 866 = 1 - 1/10 - 1/30$. (It is easy to use these factors for 12 terms without tables.)

5. Now consider the change from S_1 and C_1 to S_2 and C_2. We have to substitute 86 for 81. The difference is +5 as in Table II.; which is to be multiplied by the approximate factors 0 and +1. This is secured in Tables III. and IV. The next substitution involves forming the difference $61 - 83 = -22$, and multiplying it as in Tables III. and IV. It will be clear that all we have now to do is to form successive sums thus :

Top row of Table III. $= S_1$
Add next number $0 = S_2$
,, ,, ,, $-11 = S_3$
,, ,, ,, $-3 = S_4$

We thus get values of S and C as in Table V.

TABLE V.

	S.	C.	R.	θ.
1749–1760	+ 4	+148	25	2°
1750–1761	+ 4	+153	26	1
1751–1762	− 7	+134	22	357
1752–1763	−10	+132	22	356

We can then form R and θ where

$$36R^2 = S^2 + C^2 \quad \text{and} \quad \tan\theta = S/C,$$

R being then the amplitude of the assumed periodicity of 12 years, and θ its phase.

6. Now it is clear that this is a gradual and continuous process which cannot *introduce* any discontinuities; so that R and θ should change gradually and continuously if the observations are continuous. Clearly:

(*a*) If there happens to be a single well-marked periodicity of the period selected (in this case 12 years), then R and θ will remain nearly constant.

(*b*) If there is a single well-marked periodicity of period greater than 12 years, the values of θ will steadily increase: if the period is less than 12 years, the values of θ will steadily decrease. The values of R may oscillate, owing to the error in assumed period, but will remain nearly constant.

(*c*) If there is more than one periodicity, but all the periodicities are of a permanent character, there will be "interferences" which will give a wavy line for θ. But there should nevertheless be a distinct regularity about this line: there should be no abrupt changes. The numerical process cannot introduce them, and permanent periodicities would not cause them. If there are abrupt changes, then they must represent discontinuities in the observations.

7. This being premised, we can interpret Table VI., which shows the changes in θ, and it will be seen at once that there are abrupt changes in the neighbourhood of the horizontal divisions. A diagram shows them very plainly, but it is scarcely necessary to print a diagram, since the changes are obvious enough in the figures, from comparison with simple linear formulæ given under the heading θ_c. It will be seen that each formula holds with considerable accuracy for a number of years and then fails. The breaks are near the dates

1766, 1796, 1838, 1868, 1895,

which are sufficiently close to those of Leonid perihelia to suggest a *vera causa*. We will consequently make a suitable hypothesis in a moment, but first we may give the formulæ used for θ_c, with the corresponding periods implied for Sun-spot variation.

TABLE VI.

Years.	θ_0.	θ_c.	$\theta_0 - \theta_c$.	Years.	θ_0.	θ_c.	$\theta_0 - \theta_c$.
	°	°	°		°	°	°
1749-60	2	4	− 2	1759-70	277	...	...
61	1	1	0	71	280	285	− 5
62	357	358	− 1	72	280	277	+ 3
63	356	355	+ 1	73	266	269	− 3
64	352	352	0	74	260	260	0
1754-65	347	349	− 2	75	257	252	+ 5
66	347	346	+ 1	76	258	244	+14
67	344	343	+ 1	77	246	235	+11
68	314	...	...	78	212	227	−15
69	280	...	...	79	204	219	−15

TABLE VI.—*continued.*

Years.	θ_0.	θ_c.	$\theta_0 - \theta_c$.	Years.	θ_0.	θ_c.	$\theta_0 - \theta_c$.
	°	°	°		°	°	°
1769–80	204	210	− 6	1808–19	211	209	+ 2
81	200	202	− 2	1809–20	212	210	+ 2
82	189	194	− 5	21	212	211	+ 1
83	179	185	− 6	22	213	212	+ 1
84	175	177	− 2	23	213	213	0
85	174	169	+ 5	24	213	214	− 1
86	171	160	+11	25	212	215	3
87	150	152	− 2	26	204	216	−12
88	135	144	9	27	202	217	−15
89	134	135	− 1	28	203	218	−15
1779–90	132	127	+ 5	29	207	219	−12
91	124	119	+ 5	1819–30	216	220	− 4
92	119	110	+ 9	31	221	221	0
93	116	102	+14	32	223	222	+ 1
94	116	94	+22	33	223	223	0
95	116	85	+31	34	225	224	+ 1
96	116	77	+39	35	220	225	− 5
97	119	...	...	36	181	226	−45
98	132	188	−56	37	151	227	−76
99	154	189	−35	38	151	228	−77
1789–1800	173	190	−17	39	155	...	...
01	187	191	− 4	1829–40	155	145	+10
02	195	192	+ 3	41	150	144	+ 6
03	194	193	+ 1	42	142	143	− 1
04	197	194	+ 3	43	141	142	− 1
05	198	195	+ 3	44	141	141	0
06	192	196	− 4	45	138	140	− 2
07	189	197	− 8	46	140	139	+ 1
08	188	198	−10	47	137	138	− 1
09	187	199	−12	48	137	137	0
1799–1810	187	200	−13	49	126	136	−10
11	187	201	−14	1839–50	122	135	−13
12	191	202	−11	51	122	134	−12
13	198	203	− 5	52	120	133	13
14	205	204	+ 1	53	120	132	−12
15	205	205	0	54	120	131	−11
16	205	206	− 1	55	120	130	−10
17	205	207	− 2	56	118	129	−11
18	205	208	− 3	57	121	128	− 7

TABLE VI.—*continued.*

Years.	θ_0.	θ_c.	$\theta_0 - \theta_c$.	Years.	θ_0.	θ_c.	$\theta_0 - \theta_c$.
1847–58	122°	127°	− 5°	1874–85	86°	79°	+ 7°
59	122	126	− 4	86	80	79	+ 1
1849–60	125	125	0	87	79	79	0
61	126	124	+ 2	88	78	79	− 1
62	124	123	+ 1	89	77	79	− 2
63	124	122	+ 2	1879–90	77	79	− 2
64	122	121	+ 1	91	70	79	− 9
65	118	120	− 2	92	55	79	−22
66	117	119	− 2	93	48	79	−31
67	117	118	− 1	94	48	79	−31
68	113	117	− 4	95	47	...	...
69	101	116	−15	96	44	28	+16
1859–70	83	115	−32	97	34	31	+ 3
71	81	79	+ 2	98	34	34	0
72	81	79	+ 2	99	34	37	− 3
73	81	79	+ 2	1889–1900	34	40	− 6
74	78	79	− 1	01	35	43	− 8
75	73	79	− 6	02	35	46	−11
76	68	79	−11	03	39	49	−10
77	68	79	−11	04	49	52	− 3
78	68	79	−11	05	55	55	0
79	68	79	−11	06	59	58	+ 1
1869–80	70	79	− 9	07	60	61	− 1
81	72	79	− 7	08	61	64	− 3
82	83	79	+ 4	09	68	67	+ 1
83	90	79	+11	1899–1910	68	70	− 2
84	90	79	+11	1900–11	62	73	−11

(α) For the years 1749 to 1767 the adopted formula is

$$\theta_c = \text{const.} - 3^\circ t,$$

where t is measured in years. The phase thus decreases 3° each year, and the true period is

$$12/(1 + 3/30) = 10{\cdot}9 \text{ years.}$$

(β) For the years 1760 to 1796 the adopted formula is

$$\theta_c = \text{const.} - 8^\circ{\cdot}3\, t.$$

The period is $12/(1 + 8{\cdot}3/30) = 9{\cdot}37$ years.

(γ) From 1787 to 1838 the adopted formula is

$$\theta_c = \text{const.} + 1^\circ t.$$

The period is $12/(1 - 1/30) = 12{\cdot}4$ years.

(δ) From 1829 to 1870 the adopted formula is

$$\theta_c = \text{const.} - 1^\circ t.$$

The period is $12(1 + 1/30) = 11{\cdot}6$ years.

(ε) From 1860 to 1894 the adopted formula is

$$\theta_c = \text{const.}$$

The period is therefore 12·0 years.

(ζ) From 1890 to 1911 the adopted formula is

$$\theta_c = \text{const.} + 3^\circ t.$$

The period is $12/(1 - 3/30) = 13{\cdot}3$ years.

8. Now, armed with this knowledge, we can recur to the original Wolf numbers and arrange them in the proper periods as in Table VII., when it will be seen that they fit very closely between the disturbances.

TABLE VII.

Wolf's Numbers arranged in Suitable Periods.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1749	81	83	48	48	31	12	10	10	32	48	54	63	
1761	86	61	45	36	21	11	38	70	106				
1770	101	82	66	35	31	7	20	92	154				
1779	126	85	68	38	23	10	24	83	132				
1788	131	118	90	67	60	47	41	21	16	6	4	7	
1800	14	34	45	43	48	42	28	10	8	2	0	1	5
1813	12	14	35	46	41	30	24	16	7	4	2	8	
1825	17	39	50	62	67	71	48	28	8	13	57		
1836	122	138	103	86	63	37	24	11	15	40	62	98	
1848	124	96	66	65	54	39	21	7	4	23	55		
1859	94	96	77	59	44	47	30	16	7	37	74	139	
1871	111	102	66	45	17	11	12	3	6	32	54		
1882	60	64	64	52	25	13	7	6	7	36	73	85	
1894	78	64	42	26	27	12	10	3	5	24	42		
1905	64	54	62	49	44	19	6						

9. As a concession to the sceptical, the uniform period 11·5 years has been adopted from 1834 to the present time; and the numbers fit fairly well. But it will be noticed that Table VI. gives three separate periods in this interval, viz. 11·6 years, 12·0 years, and 10·8 years. We shall presently consider the discontinuities more in detail, but we must first formulate some physical hypothesis to compare with the numerical facts.

Hypothesis of a Sun-spot Swarm (SSS).

10. It seems clear that it is not the Leonids themselves which are responsible for Sun-spots. Their advent upsets the regularity

of the Sun-spot cycle, which must be due to something periodic in about eleven years. Moreover, the perihelion distance of the Leonids is about an astronomical unit, so that unless the swarm is extravagantly large, its borders cannot reach the Sun.

11. We postulate, therefore, another swarm revolving in about eleven years, with a very small perihelion distance, comparable with the Sun's radius. The semiaxis major of this Sun-spot swarm (which we may denote for brevity by SSS) is about $\sqrt[3]{121}$ astronomical units or nearly 5 units. If then

$$a(1-e) = 0{\cdot}005,$$

we have $$e = 1 - {\cdot}001,$$

and $$b = 5(1 - 1 + {\cdot}002) = {\cdot}01.$$

On any diagram of an ordinary size the orbit would be practically a straight line, with one end at the Sun and the other at a distance 10 from him, which is approximately the distance of Saturn. (We shall see the significance of this fact presently.) The velocity at the Sun end is, of course, large, but that at the other end will be small. We can readily assign approximate positions of the swarm in its orbit from the well-known formula $(a - e\sin a)$, in which it will suffice to put $e = 1$. This quantity varies from 0 to π; so that if we denote the semiperiod by unity, then $(a - e\sin a)/\pi$ will give the fractions of a semiperiod corresponding to the value a. Denoting the major axis also by unity, $(1 - \cos a)/2$ will denote the fraction of its distance from perihelion at this time. Now when

$a =$	0°	30°	60°	90°	120°	150°	180°
$(a - \sin a)/\pi =$	·00	·01	·06	·18	·39	·67	1·00
$(1 - \cos a)/2 =$	·00	·07	·25	·50	·75	·93	1·00

and from a diagram we easily deduce

Time =	0·0	0·10	0·20	0·30	0·40	0·50	0·60	0·70	0·80	0·90	1·00
Distance =	0·0	0·38	0·54	0·66	0·76	0·85	0·91	0·95	0·97	0·99	1·00

the time being expressed as a fraction of the semiperiod, and the distance as a fraction of the major axis. This shows the lingering of the swarm near aphelion. In this position it will be sensitive to perturbation, and it is therefore natural to assume that the aphelion lies near the Leonid orbit, and that as the Leonids come by they cause the disturbances noted. Pivoting one end of the line on the Sun, there are two directions in which the other end will lie near the Leonid orbit, one near the incoming and one near the outgoing Leonids. The facts suggest the former as the right one, and we will now consider the crises in detail, beginning for simplicity with that of 1833.

The Leonids in 1833.

12. The main Sun-spot swarm was in perihelion in 1816–7, and thus in aphelion about $6\frac{1}{4}$ years later, say in 1823, having

probably encountered no Leonids on its passage out. But on the return voyage the Leonids had arrived and disturbance ensued. We shall adopt the supposition that aphelion A lay beyond the point B at which the two orbits cross. Hence from A to B the Leonids will accelerate the SS swarm, but then their attraction will act in the reverse direction, until a point C is reached, on the side near the Sun, at which the work of acceleration is undone. The path AC has, however, been traversed more rapidly than would have been the case had the Leonids not been there. To visualise the nature of the result qualitatively let us suppose an extreme case—that the path AC is traversed in no time at all. The SS swarm is then transferred from A to C, and will have the same velocity at C that it had at A, viz. the aphelion velocity. Thus C becomes a new aphelion, much nearer the Sun; and the period is shortened.

13. This particular case illustrates the general nature of the action in all cases. But it may be well, before going further, to state this general character more explicitly.

General Statement of Perturbative Action.

(*a*) If the SS swarm crosses the Leonids, then we can always imagine two points (A and C) on opposite sides of the Leonids (L) such that the Leonids accelerate the motion from A to L, and retard it equally from L to C. The velocity is thus left ultimately unaffected. But the path A to C has been described in less time, because at every point of it the velocity has been greater than it would have been had the Leonids not been there.

(*b*) If the crossing is on the way to perihelion, then the Sun is increasing the velocity from A to C. But owing to the presence of the Leonids, it is allowed less time than usual for this increase. Hence the SSS arrives at C with less velocity than usual: and the period is shortened.

(*c*) On the other hand, if the crossing is on the way out from perihelion, the Sun is decreasing the velocity from A to C: but is again allowed less time than usual to do so. Hence the SSS arrives at C with a velocity greater than usual, shoots further afield, and the period is lengthened.

(*d*) But we have still to consider the case when the SSS would not normally cross the Leonids. It is easy to see that it may in this case be *pulled across*. Suppose the position of aphelion is close to the Leonids. Instead of the outward velocity dying down completely, it may be continued enough to bring the SSS *very* close, when the attraction of the Leonids becomes very great—enough to pull the SSS through itself, even against the Sun's attraction. If the Leonids remained stationary, this effect would be undone on the return journey: but the SSS spends some years in aphelion, and meantime the Leonids may have passed away. The period would thus remain considerably and permanently lengthened, and it seems probable that this happened at the 1800 return, which we shall presently consider.

(*e*) Further remarks on this kind of action will be found in my papers on Stationary Radiants (*M.N.*, lix. p. 140, and lxii. p. 448), though when we have a solid body like the Earth concerned the situation is in many ways different. The case of two meteor swarms, allowing of the free passage of one through the other, introduces the notion of a modified law of gravity for the actual passage. So long as the two swarms are clear of one another, we shall have approximately the inverse square of the distance. But when they interpenetrate we must have some law decreasing with the distance, the exact nature of which depends, of course, on the unknown structure of the two swarms.

The Leonids in 1799.

14. Previous to the disturbance of 1799, the SSS was revolving in 9 years. It was at perihelion in 1787–88, and met with no disturbance on the way in. But the next return in 1802–05 shows a considerable lengthening of the period. My tentative explanation of this fact is that on arrival at aphelion in 1792 the SSS was near enough to the Leonids to be *pulled across* as above described. Aphelion distance for an SSS revolving in 9 years is about 8·5 astronomical units: and at this distance the Leonids would be about 2 years from perihelion. Hence the part of the Leonids which must have produced this effect would reach perihelion about 1794. This is rather earlier than our Earthly experiences would suggest, but even on our Earth we had fireballs as early as 1792: and there seems nothing impossible in the swarm being strong enough even in this advance guard to produce the above effect near aphelion, which is a sensitive part of the orbit. We must now make as good an estimate as we can of the circumstances of the return journey. We can only infer the time of leaving aphelion from that of the subsequent rounds. We may put the perihelion (maximum) at 1803·5, and the period as $12\frac{1}{3}$ years. Hence aphelion must have been left in $1803{\cdot}5 - 6{\cdot}2 = 1797{\cdot}3$. To this we must add the unknown time from aphelion to the Leonid orbit, which may be a year or even two -depending on how far aphelion was "pulled across." Let us say 1·7 years as a fair allowance, making the crossing point 1799·0. If the Leonids were then all gone by, then their perihelion passage must have been all over 2 years later, in 1801·0. This again is a little early for the end of the shower according to our terrestrial experience. But our knowledge of the Leonids is not very precise, and so far as I can ascertain the perihelion passage might be earlier than usual by a few years, owing to planetary perturbations. We must be content to leave the matter there for the present, remarking:

(1) That an exceptionally early passage of the Leonids at this return would fit all the facts. (See end of § 46.)

(2) That there is nothing against an effect having been produced on the return journey, so long as it was smaller than the

outward effect. All we know is that the period was considerably lengthened between 1788 and 1804.

The Leonids in 1766.

15. We now come back to the earliest return on which Wolf's numbers can give us any information. If we have rightly diagnosed the return of 1800, the SSS aphelion between 1770 and 1788 fell short of the Leonid orbit, though pretty close to it: and was carried across after 1788. There would seem to have been a reverse change about 1766. The SSS was at perihelion about 1749 (or earlier?—we cannot say) and got out to aphelion beyond the Leonid orbit in 1755 without mishap. But during its loitering at aphelion the Leonids came by, carried the aphelion well over to the side near the Sun, and left the period shortened. The SSS reached perihelion under the new conditions in 1760–61, and aphelion about 1765. Either the Leonids were then chiefly gone by, or such as were left were not strong enough to undo more than very partially the work done earlier. The net result was a considerable shortening of the period. It is noteworthy that here also an early return of the Leonids would suit the facts the better.

The Leonids of 1866.

16. We now come to the 1866 return, and it is obvious from Table VII. that the chief fact to be explained is the negligible effect on the period. The SSS was at perihelion in 1859–60, and judging from the accordance with the previous maximum it encountered no appreciable disturbance on its way from the previous aphelion, which took place about 1854. But on its way out to the aphelion of 1865–66 it must have crossed the Leonid orbit if our previous suppositions are correct. The crossing is near aphelion, but the circumstances are different from those of previous aphelion passages. When the period was 9 years, the aphelion lay on the Sun side of the Leonid orbit: when it was 12 or 13 years on the further side. In 1866 the period is intermediate, and aphelion is due close to the Leonid orbit, just about the time of passage. There will thus be a rough symmetry about the circumstances. The action before aphelion will be to lengthen the period, after aphelion to shorten it, and the net result is apparently nearly zero, so far as the main portion of the SSS is concerned. But we can distinctly trace the effect of the Leonids by considering another part of the SSS, especially that in perihelion in the years 1863–65, represented by the Wolf numbers 44, 47, 30. It will be seen that these numbers do not repeat in the line below as they should if there had been no disturbance; and that minimum is reached earlier and maintained longer in 1876 than in 1866. This portion 44–47–30 is due at aphelion several years later than the main swarm: it thus encounters the first half of the perturbation but not the second, or at any rate the first half more

than the second. Now the first half is the lengthening of the period; and hence for the portion 44–47–30 the period is lengthened, and when it was due at perihelion it had not returned. *When* it returned is not easy to say with our imperfect knowledge, but we shall presently find evidence of a subsidiary long period which may have a bearing on the question.

17. We should also consider the portion on the other side of the main swarm: say the portion represented by Wolf's numbers 23–55 for years 1857–58. This should more or less escape the first part of the perturbation (lengthening), but encounter the second in greater degree than the main swarm. Hence its period should be shortened. It should get back early and make an earlier or more rapid rise to maximum. It will be seen that this is the case, the numbers in the column below being larger. The effect is still more noticeable if we rewrite the series so as to allow for the half years thus:—

54	39	21	7	4	23	55	94	96
47	30	16	7	37	74	139	111	

where it will be seen that the lower line alternates properly with the upper until we reach the rise to maximum, when the lower line anticipates the upper.

The Leonids of 1899.

18. Finally we have to consider the last return when the Leonids failed to strike our Earth, and may for all we know have been considerably perturbed. The SSS apparently arrived at maximum in 1893 without serious mishap, although there is again a more rapid rise, indicating that it may have been hurried up on its way from aphelion. It was due at aphelion again in 1899, a year or two *after* the Leonids probably passed on their way to perihelion. Hence, though the perturbation would still be of a dual nature as in 1866, first lengthening and then shortening the period, the first half would predominate, and the period would be lengthened on the whole. This is shown both by the slower rise to maximum and by the curious recrudescence of maximum in 1907, due no doubt to the specially late arrival of those members of the swarm which had their period most lengthened.

Hypothesis of no Disturbance.

19. On a hasty reading, or even a careful *first* reading, the above argument may seem rather elaborate and fanciful. It would be much more satisfactory if even a rough numerical process could be devised, and I do not think it is impossible to do so. But it would certainly involve much work of a "trial and error" kind, in our ignorance of the numerous constants concerned: and the above argument, when followed step by step, seems to be sound.

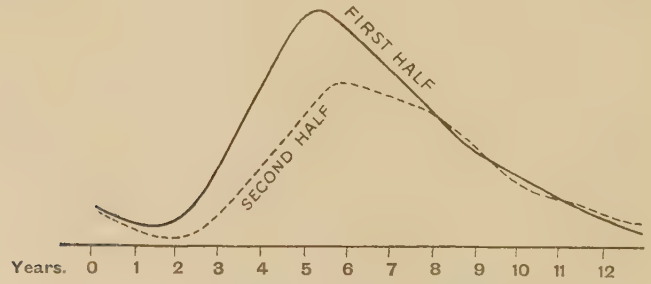
20. But an independent method of examination, which may

commend itself more readily to those who feel a natural scepticism, will now be presented.

21. Since the 1833 return, the period has been very nearly uniform at $11\frac{1}{2}$ years. Let us assume it uniform, and see how far we can account for the facts. Perhaps, after all, the early Wolf numbers are faulty?

22. Now in the first place we find that there is not a satisfactorily uniform period of $11\frac{1}{2}$ years, but rather a double period of 23 years, the halves of which are not quite symmetrical. The evidence for this can be clearly presented by writing the Wolf numbers in rows of 23. This is rather a long row for a printed page: but we can write two sets of 12 years thus, repeating one year twice:—

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1842	24	11	15	40	62	98	124	96	66	65	54	39
1865	30	16	7	37	74	139	111	102	66	45	17	11
1888	7	6	7	36	73	85	78	64	42	26	27	12
Mean	20	11	10	38	70	107	104	87	58	45	33	21
Interpolated		16	11	24	54	88	106	96	72	52	39	27
Excess		+2	+7	+19	+28	+38	+33	+25	+4	−1	+1	+1
1853	39	21	7	4	23	55	94	96	77	59	44	47
1876	11	12	3	6	32	54	60	64	64	52	25	13
1899	12	10	3	5	24	42	64	54	62	49	44	19
Mean	21	14	4	5	26	50	73	71	68	53	38	26
Interpolated		18	9	4	16	39	62	72	70	60	48	32
Excess		−2	−2	−6	−22	−31	−45	−32	−17	+2	+3	−1



Alternate halves of 23-year cycle compared.

The mean numbers for the first set should include those for the next set between them. The interpolations in each set are shown compared with the other, and it will be seen that the differences are considerable near maximum, giving the impression of a second swarm which returns once every 23 years only. But whatever the cause, we must clearly recognise this difference in any further analysis.

23. Secondly, even though the period may be constant, the intensity of the Sun-spots is gradually dying out in the years 1836 to 1910. Let us take the three sets of double maxima as follows:—

Wolf's Nos. at Maximum.

		Total.	Log.
1st Cycle	$+122+138+103$ / $+98+124+96+66$	$=747$	2·873
2nd Cycle	$94+96+77$ / $+139+111+102+66$	$=685$	2·836
3rd Cycle	$60+64+64$ / $+85+78+64+42$	$=457$	2·660

24. If we regard the drop in the log (total) as uniform, it is at the rate of 0·213 in 46 years, or 0·00463 per year. A smooth curve was drawn for the mean of all the cycles, the numbers read from it, and the logs of them taken. The above quantity was then added to successive years, so as to allow for uniform decadence, and in this way the logs in the second column of Table VIII. were formed, from which the numbers in the column C were obtained. The residuals O−C thus represent the best we can do for the hypothesis of uniformity, without abrupt changes.

TABLE VIII.

Theoretical Curve, on Hypothesis of Uniform Action.

Year.	log.	C.	O.	O−C.	Year.	log.	C.	O.	O−C.
1832	1·20	16	28	+12	1852	1·59	39	54	+15
33	0·85	7	8	+ 1	53	1·43	27	39	+12
34	1·45	28	13	−15	54	1·26	18	21	+ 3
35	1·85	71	57	−14	55	1·10	13	7	− 6
36	2·06	115	122	+ 7	56	0·75	6	4	− 2
37	2·09	123	138	+15	57	1·35	22	23	+ 1
38	2·01	102	103	+ 1	58	1·74	55	55	0
39	1·92	83	86	+ 3	59	1·96	91	94	+ 3
1840	1·75	56	63	+ 7	1860	1·99	98	96	− 2
41	1·54	35	37	+ 2	61	1·91	81	77	− 4
42	1·26	18	24	+ 6	62	1·81	65	59	− 6
43	1·01	10	11	+ 1	63	1·65	45	44	− 1
44	1·01	10	15	+ 5	64	1·43	27	47	+20
45	1·68	48	40	− 8	65	1·16	14	30	+16
46	1·92	83	62	−21	66	0·90	8	16	+ 8
47	2·06	115	98	−17	67	0·90	8	7	− 1
48	2·11	129	124	− 5	68	1·57	37	37	0
49	2·00	100	96	− 4	69	1·81	65	74	+ 9
1850	1·81	65	66	+ 1	1870	1·95	89	139	+50
51	1·78	60	65	+ 5	71	2·00	100	111	+11

TABLE VIII.— *Theoretical Curve, on Hypothesis of Uniform Action*—continued.

Year.	log.	C.	O.	O−C.	Year.	log.	C.	O.	O−C.
1872	1·89	78	102	+24	1893	1·84	69	85	+16
73	1·69	49	66	+17	94	1·89	78	78	0
74	1·67	47	45	− 2	95	1·79	62	64	+ 2
75	1·48	30	17	−13	96	1·59	39	42	+ 3
76	1·31	20	11	− 9	97	1·56	36	26	−10
77	1·15	14	12	− 2	98	1·37	23	27	+ 4
78	0·98	10	3	− 7	99	1·21	16	12	− 4
79	0·63	4	6	+ 2	1900	1·04	11	10	− 1
1880	1·23	17	32	+15	01	0·88	8	3	− 5
81	1·63	43	54	+11	02	0·53	3	5	+ 2
82	1·84	69	60	− 9	03	1·13	13	24	+11
83	1·87	74	64	−10	04	1·52	33	42	+ 9
84	1·79	62	64	+ 2	05	1·74	55	64	+ 9
85	1·70	50	52	+ 2	06	1·77	59	54	− 5
86	1·52	33	25	− 8	07	1·69	49	62	+13
87	1·32	21	13	− 8	08	1·60	40	49	+ 9
88	1·04	11	7	− 4	09	1·43	27	44	+17
89	0·79	6	6	0	1910	1·22	17	19	+ 2
1890	0·79	6	7	+ 1	11	0·94	9	6	− 3
91	1·46	29	36	+ 7	12	0·69	5		
92	1·70	50	73	+23					

25. It will be seen that there is a distinct jump at 1864. There is a bigger one in 1870, but this can be referred to a special cause, as will be seen later (see § 34). There is no serious jump near 1900, but that at 1903 is sensible; and on the whole if this were all, one might accept the hypothesis of steady variation, with decadence of coefficient, for this part of the series. The independent evidence which follows, however, is too strong to be overlooked.

26. There is one more point. If we arrange these residuals in 14 years we find a distinct periodicity as below. (The numbers in brackets have been omitted as exceptional, for which good reason will be given later.) Now this seems to represent the survival of part of the swarm which revolved in a long period, up to 1830. Hence the periodicity is certainly not *quite* simple even in the period 1830–1911.

+12	+ 1	−15	−14	+ 7	+15	+ 1	+ 3	+7	+ 2	+ 6	+ 1	+ 5	− 8
−21	−17	− 5	− 4	+ 1	+ 5	+15	+12	+3	− 6	− 2	+ 1	0	+ 3
− 2	− 4	− 6	− 1	+20	+16	+ 8	− 1	0	+ 9	(+50	+11	+24	+17)
− 2	−13	− 9	− 2	− 7	+ 2	+15	+11	−9	−10	+ 2	+ 2	− 8	− 8
− 4	0	+ 1	+ 7	+23	+16	0	+ 2	+3	−10	+ 4	− 4	− 1	− 5
− 5	− 7	− 7	− 5	+ 9	+11	+ 8	+ 5	+1	− 3	+ 2	0	− 1	− 41

Hypothesis of Formation of SSS *by Saturn.*

27. There are some features of the above hypothesis which seem curiously artificial unless they can be related to a *vera causa*, especially the existence of a swarm (the SSS) moving in an orbit in the same plane as the Leonids and with an aphelion so close to their actual path. If the SSS, however, is a detached portion of the Leonids, this would be explained. The aphelion then becomes the relic of the point of detachment. Is there any planet which could be responsible for the detachment? The period of the SSS varies between 9 and 13 years, the mean value being near 11 years. Since $11^{\frac{2}{3}} = 4{\cdot}9$, the major axis is $9{\cdot}8$: and since the perihelion distance is very small, the aphelion must be close to the orbit of Saturn. It is therefore natural to inquire whether Saturn could have detached a portion of the swarm at some near approach to the Leonids. The general nature of the perturbation must have been to leave the detached portion, after the passage of Saturn, with very small velocity (say zero velocity for simplicity), so that it fell almost directly into the Sun. We will make a rough calculation of the possibilities.

28. Let the velocity of Saturn in its orbit be V.

Then $$V^2 = \mu\left(\frac{2}{10} - \frac{1}{10}\right) = \mu/10,$$

since both the distance r and semimajor axis a of Saturn's orbit may be taken as 10. If v be the velocity of the Leonids

$$v^2 = \mu\left(\frac{2}{10} - \frac{1}{22{\cdot}4}\right) = 1{\cdot}554\ V^2,$$

$$\text{or } v = 1{\cdot}25\ V.$$

Now the Leonids cross the orbit of Saturn at an angle of about 60°: call it α for the present. Their velocity relative to Saturn is thus

$$v\cos\alpha + V \text{ along the orbit}$$

and $$v\sin\alpha \text{ perpendicular to it}$$

total relative velocity $$w^2 = v^2 + V^2 + 2vV\cos\alpha.$$

The encounter with Saturn will leave this total relative velocity unchanged in amount, but altered in direction. The alteration in direction may be considerable, as in the case of the encounter with Uranus in A.D. 126 which swept the Leonids into their present orbit. If after the encounter the Leonids are nearly stationary with respect to the Sun, then their velocity relative to Saturn is approximately V. But we cannot have

$$V^2 = v^2 + V^2 + 2vV\cos\alpha.$$

Indeed the two sides of the equation must be very different. We have, of course, neglected important quantities, such as the curvature of both orbits; and the Sun's attraction. But the disparity of the two sides of the equation is so great that it seems

unlikely that a more exact calculation would yield a sufficiently small value of the velocity-after-encounter to make it probable that the detachment was purely gravitational.

29. Could it have taken place in any other way? Leonids which struck the ball of Saturn would probably be lost from the swarm altogether: but what of those which encountered the rings? The rings are known to be loose in structure, and it is conceivable that they might partially arrest a certain number of meteors in some way, leaving them with such a velocity as is requisite.

30. This, however, is mere speculation, and will only be worth attention if an occasion can be identified on which Saturn actually passed through the swarm. Now the swarm was swept into its present orbit by Uranus in A.D. 126, which gives one limit for the possible opportunities. On the other hand, a Sun-spot was recorded in the Chinese annals in A.D. 188 (see *Observatory*, vol. xii. p. 218), though this was an isolated instance. A century later (A.D. 299–304) many were recorded. The time for a suitable encounter with Saturn is thus limited to a couple of centuries. From Mr. Denning's diagram (*Observatory*, XX. p. 200), it will be seen that the longitude of Saturn at the crossing in A.D. 126 was about 170°, which was his longitude in 1891·7. Taking the period of Saturn as 29·457 years, 58 revolutions occupy 1708·5 years, which takes us back to A.D. 183·2. Saturn will thus be in the node in

A.D. 183·2, 212·7, 242·1, 271·6, 301·0, 330·5, 360·0, etc.

31. Now the period of the Leonids is somewhat uncertain. If we carry back $33\frac{1}{4}$ years (the period assumed by Adams) from A.D. 902, we come to 204 as a perihelion passage: and the crossing of Saturn's orbit would be about 2 years earlier than this, say 202 A.D. Hence we have the Leonids in the node in

A.D. 169, 202, 235, 268, 302, 335, etc.

32. The year 301 seems most promising for an encounter with Saturn, though a slight alteration of the Leonid period would make 271 more likely, as further evidence will support. It seems clear that we could not have an encounter between A.D. 126 and A.D. 188, so that the Sun-spot of A.D. 188 cannot have been due to Saturn in any way. But now remark that owing to the difference of nearly 4 years between the period of Saturn and that of the Leonids such an encounter would not recur for about 266 years, since

$$8 \times 33\tfrac{1}{4} = 266{\cdot}0 \text{ years}$$
$$9 \times 29{\cdot}457 = 265{\cdot}113 \text{ years},$$

which must be increased to 265·44 years in order to let Saturn catch up the revolving orbit of the Leonids. Hence it is a distinct point in our favour that such an encounter should take place between the date of the introduction of the Leonids and the second recorded Sun-spot.

TABLE IX.

Mr. Hirayama's Chinese Sun-spots (*Observatory*, xii. p. 218).

Date.	Description.
188, Feb. 14	The colour of the Sun reddish-yellow. Fleckle in the Sun (bird-shaped).
299, ,, 17	Fleckle in the Sun (bird-shaped).
301, Jan. 18	Fleckle in the Sun.
,, Oct. 21	,, ,,
302, Dec. 7	,, ,,
304, ,, 15	,, ,,
311, Apr. 8	Flying-bird in Sun.
321, May 7	Fleckle in the Sun.
322, Nov. 7	,, ,,
342, Mar. 8	,, ,,
354, Nov. 8	,, ,, (like an egg).
355, Apr. 5	Fleckle in the Sun.
359, May 10	,, ,,
369, Nov. ?	,, ,,
370, Mar. 30	,, ,, (like an apple).
373, Jan. 28	Fleckle in the Sun.
,, Dec. 26	,, ,, (like an apple).
374, Apr. 7	2 fleckles in the Sun.
375, Jan. 11	Fleckle in the Sun.
388, Apr. 3	,, ,,
395, Dec. 13	,, ,,
400, ,, 7	,, ,,
499, Jan. 31	3 fleckles in the Sun.
826, Mar. ?	Fleckle.
832, Apr. 25	,,
837, Dec. 25	,,
842, Jan. 3	,,
864, Feb. ?	,, (egg-shaped).
874, Jan. ?	,, (flying-bird-shaped).
974, Mar. 6	2 fleckles in the Sun.
1005, Jan. 10	2 fleckles.
1077, Mar. 13	Fleckle like apple.
,, ,, 27	Continuously seen.
1078, Mar. 17	Fleckle in the Sun.
,, Apr. 4	,, ,,
1079, Jan. 17	,, ,, (apple-shaped).
,, Mar. 25	
1103, July 8	Great diminution in the light in the day-time.
1104, Nov. 1?	Fleckle in the Sun.
1112, May 9	,, ,,
1118, Dec. 24	,, ,,
1120, Feb. 25	,, ,,
,, June 14	,, ,,
1122, Jan. 17	,, ,,
1129, Mar. 29	,, ,,
,, Apr. 21	,, ,,
1131, Mar. 19	,, ,,
1136, Dec. 1	,, ,,
,, ,, 5	,, ,,
1137, Mar. 8	,, ,,
,, May 13	,, ,,
1138, Mar. 24	,, ,,
,, Nov. 13	,, ,,
1145, Oct. 0	,, ,, (motion observed).
,, ,, 1	
1185, Feb. 17	Fleckle in the Sun.
,, ,, 21	,, ,,
,, Mar. 4	
1186, May 31	,, ,,
1193, Dec. 10	,, ,,
,, ,, 19	,, ,,
1200, Sept. 28	,, ,,
,, Oct. 3	,, ,,
1201, Jan. 16	,, ,,
,, Feb. 5	,, ,,
1202, Dec. 26	,, ,,
1203, Jan. 8	,, ,,
1205, May 11	,, ,,

TABLE IX.—*Mr. Hirayama's Chinese Sun-spots* (*Observatory*, xii. p. 218)—continued.

Date.	Description.
1238, Dec. 12	Fleckle in the Sun.
1276, Feb. 24	,, ,, (flying-bird-shaped).
1370, Jan. 9	Fleckle in the Sun. (flying-bird-shaped).
,, Oct. 29	Fleckle in the Sun.
,, Nov. 15	,, ,,
1371, Apr. 8	,, ,,
,, June 22	,, ,,
,, Nov. 14	,, ,,
1372, Feb. 15	,, ,,
,, Apr. 9	,, ,,
,, Jan. 10	,, ,,
,, Sept. 1	,, ,,
1373, Nov. 23	,, ,,
1374, Apr. 2	Fleckle in the Sun.
,, ,, 6	,, ,,
1375, Mar. 29	,, ,,
,, Oct. 28	,, ,,
1376, Jan. 25	
1381, Mar. 25–29	,, ,,
1382, ,, 29	,, ,,
1383, Jan. 18	,, ,,
1616, Oct. 10	Dark light in the Sun.
1618, Aug. 17	Fleckle in the Sun.
,, ,, 19	,, ,,
1624, Mar. 15	,, ,,
,, May 26	,, ,,
1638, Dec. 10	,, ,,

33. But there is much more than this in our favour. When the encounter recurs after 265·44 years, does Saturn again detach some Leonids to swell the SSS? The circumstances may not be *quite* so favourable, owing to the discrepancy between the above two multiples. But it is at least worth inquiring whether the Chinese records above mentioned show any such "refreshing" of the Sun-spots. The critical dates will be at or near

A.D. 301·0, 566·4, 831·9, 1097·3, 1362·8, 1628·2, 1893·7 . . .

Now the best way to present the evidence seems to be to reproduce here Mr. Hirayama's table above cited. It is some years since it was printed, and it may be useful for reference. It will be seen that the above dates are startlingly prominent, with two qualifications: firstly, 566·4 is not represented at all in the table; and secondly, the Sun-spots begin a little in advance of the critical dates. If we shift them all back one revolution of Saturn, then the critical dates are all preceded by an absence of Sun-spots and all followed by an outburst. It must be remembered that after an encounter, the detached meteors take about 5·7 years to reach the Sun. Hence we have for the critical encounters the years

271·6, 537·0, 802·5, 1067·9, 1333·4, 1598·8, 1864·3, etc.,

and for the first possible appearance of the new cycle of spots therefore the approximate dates

277, 543, 808, 1073, 1339, 1604, 1870, etc.

The Chinese annals show outbursts in

299, — 826, 1077, 1370, 1616,

while we all know the special outburst in 1870–71.

34. Did the encounter of 1864 produce any effect on Saturn? On turning up the *Monthly Notices* for 1864 the planet Saturn does not occur in the index; but in the previous volume, p. 195, there is the following:—

"*Note on the Appearance of the Dark Ring of Saturn.*
By J. Carpenter, Esq.

"During the observation of Saturn with the transit circle, on the evening of March 26, 1863, it struck me that the dark space between the ring and the ball seemed much contracted. Upon looking at it with the equatorial I found that this arose from a great increase in the brightness of the dusky ring, which appeared nearly as bright as the illuminated ring, and might easily have been mistaken for a part of it."

35. The date March 26, 1863, is sensibly earlier than that above given: but my place of Saturn was only taken from a diagram, and may be sensibly in error. It seems just worth leaving the figures as they were originally computed. But it can be now tested whether Mr. Carpenter's alertness detected an actual encounter of Saturn with the Leonids; and if so, the accuracy of the date may be of value.

36. If we now turn to the observed places of the planet himself, we find ample confirmatory evidence of a serious encounter about 1863 or 1864, as shown in the following table. The error of R.A. of Saturn in the years up to 1864 was nearly constant, but slowly increasing positively. After 1864 it began to decrease rapidly. The planet was sent reeling backwards by the blow of the Leonids. The decrease stops about 1870, but a considerable error in radius vector then develops, as a glance at the separate months shows.

Mean Errors of Saturn as shown in the Greenwich Observations.

	R.A.	Dec.		R.A.	Dec.
	s	″		s	″
1858	+0·78	− 4·1	1867	−0·19	−6·6
1859	+0·78	− 8·7	1868	−0·36	−4·2
1860	+0·82	−12·4	1869	−0·87	−2·2
1861	+0·77	−16·1	1870	−0·87	−1·1
1862	+0·86	−18·3	1871	−0·73	+0·4
1863	+0·87	−19·2	1872	−0·60	+2·0
1864	+0·66	−16·9	1873	−0·54	+4·2
1865	+0·36	−13·3	1874	−0·51	+4·9
1866	+0·12	−10·3			

37. Now these are the errors according to Bouvard's Tables, which were naturally blamed for them. The matter attracted some attention at the time, as will be seen for instance from a

paper by Mr. Dunkin "On the Observed Errors of Bouvard's Tables of Saturn" (*M.N.*, xxxix. p. 172). But when new tables were made, they naturally included the new observations and smoothed out the discontinuity. They could not entirely get rid of it, however. Dr. G. W. Hill, after doing the best he could, wrote (*Ast. Papers of Amer. Ephem. and N.A.*, vol. vii. p. 166):

"The residuals of the R.A. frequently show a systematic character, especially in the latter half of the period. However, all the efforts I have made to detect periodicity in them have led to no result. They may be attributed to one of three causes. Either some error has been committed in the theory, or some force acts on Saturn of which we know nothing, or the observations are affected with systematic errors which their combination has not completely eliminated. The last seems to have the greatest degree of probability in its favour."

The present suggestion is to adopt the second alternative; but to test it thoroughly we should discuss the observations before 1863 separately from those since. The comparison would give us the amount of disturbance of Saturn.

38. Meantime, since there seems a reasonable probability that the Leonids struck Saturn with some violence, it is natural to recur to the question how the SSS was formed. Whatever materials formed it must have been left behind nearly at rest relatively to the Sun, when both Saturn and the Leonids passed on. Now we have above considered merely the translational velocity of Saturn. When we consider his Ring, we have to add to this the velocity of revolution in the Ring. There must be parts of the Ring with velocities almost exactly equal and opposite to that of the Leonids: it is probably these parts which, meeting Leonids, come to a standstill and thereafter fall into the Sun (nearly). On the opposite side of the Ring the velocity of revolution is in the contrary sense, and therefore sympathetic with that of the Leonids. Here some Leonids may get entangled in the Ring and help to keep it going. Is it an impossibility that the preservation of the Ring as a Ring is partly due to these stimulations by the Leonids? If the effect were wholly destructive, we should have expected the Ring to show more signs of wear and tear. But of course since A.D. 126 there can only have been half a dozen encounters in all.

39. We must remember that not only do the Leonids affect Saturn, but Saturn affects the Leonids. The general nature of the perturbation has been described above:—the swarm is transferred from a point A of its orbit to a point C in less time than usual, and consequently its velocity at C is not so large as it would otherwise have been. Hence its period will *shorten*. If, for instance, $33\frac{1}{4}$ years be the mean period from 902 to 1866 (as assumed by J. C. Adams), then the present period must be less than $33\frac{1}{4}$ years, and the period in 902 must have been greater.

Perturbations of the Earth and Moon by the Leonids.

40. The foregoing theory of Sun-spots depends on the fundamental assumption that the Leonids have sufficient mass to alter the period of the SSS very considerably. If so, they must also be able to perturb appreciably other bodies near which they pass, and particularly the Earth. We need not expect such startling changes as have been noticed in the case of the SSS (alterations of period amounting to 30 per cent. of its length), because the SSS is perturbed in a very sensitive part of its orbit, viz. at aphelion, where the velocity is small. Moreover, alterations of period depend essentially on the *changes of velocity* of the body perturbed, which are great in the case of the SSS near aphelion, whereas the velocity of the Earth remains nearly constant. Hence we should expect the effect to be much smaller in the case of the Earth. How much smaller it may be quite possible to calculate, though we must first investigate the action of one swarm on another in order to estimate the mass of the Leonids. But it is simpler to look at observations at once in order to see whether there is any effect of the kind. The longest series of observations available for the purpose are those of the Moon as exhibited by Newcomb in *M.N.*, lxix. p. 164, where he finds unexplained fluctuations. One of these is of long period, and he draws a mean curve for it: but there are others of short period, and it is not a little remarkable that the places where the observation curve cuts the mean are at dates

1761, 1804, 1840, 1870, 1901,

which are reasonably close to the dates of visits of the Leonids. In his résumé of possible causes—fluctuations in the Earth's speed of rotation, tidal friction, magnetism—Newcomb does not allow for the possible action of the Leonids. He takes "it for granted that the gravitation of all known bodies has been allowed for in the comparison, and that no unknown bodies exist": but it apparently did not occur to him that meteor swarms such as the Leonids might have appreciable mass.

41. The following paragraph from Newcomb's paper is important in connection with the suggested explanation:—

"Since what is actually observed is neither the acceleration nor the speed of motion, but changes of the longitude itself, of which these quantities are respectively the second and first derivatives as to the time, it is not possible from the observations to make any approach to an accurate estimate of the accelerating or retarding forces. The most that we can say is that these varying forces are sufficient to bring about a change of annual motion amounting to between 0″·5 and 1″·0 by acting during a period of perhaps from 4 to 6 years."

42. The "action during the period of perhaps from 4 to 6 years" seem at first sight specially suitable for the Leonid action. But there is another feature of the disturbance which must be explained if we are to accept Leonid disturbance even provisionally.

Why should the action be reversed at successive returns? At successive dates in the list above given, the observation curve crosses the mean curve in opposite directions, showing that the perturbation must have been in opposite directions. A reasonable tentative explanation would be that the Moon was at opposite points of its orbit on the successive occasions. The occasions are defined by the time of year (November 14 approximately), and we know that the Moon's longitude on a given date increases by 146° approximately. Hence the position in successive years when the Earth-Moon system passes through the swarm is very different, and the Moon can only be said to have a determinate position at the encounter (say) of 1833 if the swarm has a dense centre, so that the greater part of the action takes place in one particular year. If this is the case, then 33 years later the Moon has made

$$33 \times 365{\cdot}25/27{\cdot}322 = 441{\cdot}47 \text{ revolutions.}$$

The fractional part seems to offer a reasonable chance that the alternation can be explained in this way.

43. But there is a further point. The period of the Leonids is not 33 years exactly but $33\frac{1}{4}$ (or less). We use 33 years exactly in the above calculation because it is after this interval that the Earth and Moon return to the same spot. But the dense Leonid centre which perturbs them will not have *quite* returned to its same position, and the action will thus be either greater or less. It is easy to see that this will cause the action itself to fluctuate in amount; there will be a configuration which gives it the maximum effect, followed by a dying away and a reappearance. After three or four returns (the period of the Leonids is not very exactly known) the dense nucleus will return just a year later than it did, but it will not find the Moon in the same place in its orbit. Without an exact knowledge of the Leonid period it is not easy to say where the Moon would be at the return of the dense nucleus to its original position: so that we cannot specify the length of the period during which the disturbance waxes and wanes—we can at present only say that there will be a waxing and a waning. Now no one can look at Newcomb's diagram without recognising this feature at once.

The fluctuations are large at the present time, smaller as we go back to 1760, when they almost disappear. Going back before 1660 there are fluctuations again, but these are probably owing to the imperfection of the observations which is sufficient to mask real fluctuations.

44. The long period change (250 years to 300 years) is probably due to the fractional excess of the Leonid period (say 33·2 years) over an exact number of years. Thus if the change takes place in 330 years we must assign 33·1 years as the modern value of the Leonid period.

45. But the perturbations should not be confined to the Moon. The Earth also should be affected at Leonid returns, which would make discontinuities in the observed place of the Sun. Now

in *M.N.*, liv. p. 431, the present Astronomer Royal and Mr. Thackeray carefully rediscussed the Solar observations for personality, showing that changes of observer had an appreciable effect. The series extends from 1864 to 1893 only, and therefore only includes one Leonid perihelion at the beginning of the series. For later years we can at present only use the uncorrected results from the Greenwich volumes. There was a change of tables from Leverrier to Newcomb in 1901, the effect of which is sufficiently indicated by comparisons for several years published by Dr. Downing as follows, for the mean R.A. of the Sun :—

Differences between Leverrier's and Newcomb's Tables.

	s	
For 1900	−0·030	*M.N.*, lv. 514
1901	−0·030	„ lvii. 43
1904	−0·034	„ lxii. 20
1906	−0·033	„ lxiv. 422

46. For the present purpose we may regard the effect due to the change of tables as constant and of value $-0^s\cdot032$: and the mean errors in R.A. (for the 12 monthly values) for years 1901–1911 have been *increased by* $+0^s\cdot032$ to make them continuous with previous years in the column "Uncorr." of the following table. Now as regards the 1866 Leonid return, we have scarcely enough information about previous years. There is an appearance of disturbance in 1866 and 1867; but not knowing how much of what has gone before has been taken up into the tables, we can scarcely assess its amount. In 1900 the case is different: the observations have not been used in forming the tables, and we see at once that there is a considerable change which seems to be still progressing, *i.e.* it may be an actual change in the length of the year, and not a mere discontinuity of place. It is too early yet to decide, but not too early to be on the alert.

TABLE X.

Mean Errors of Sun in R.A. Corrected for change of Tables in 1901.

Year.	Uncorr.	Corr.	Year.	Uncorr.	Corr.
	s	s		s	s
1864	+0·01	+0·02	1874	+0·05	+0·06
65	+0·02	+0·05	75	+0·06	+0·04
66	−0·01	0·00	76	+0·05	+0·04
67	−0·03	0·00	77	+0·05	+0·03
68	+0·02	+0·02	78	+0·05	+0·03
69	+0·02	+0·04	79	+0·05	+0·02
1870	+0·01	+0·04	1880	+0·05	+0·01
71	+0·03	+0·03	81	+0·05	+0·02
72	0·00	+0·02	82	+0·04	+0·02
73	+0·04	+0·04	83	+0·05	+0·02

TABLE X.—*Mean Errors of Sun in R.A. Corrected for change of Tables in* 1901—continued.

Year.	Uncorr.	Corr.	Year.	Uncorr.
	s	s		s
1884	+0·06	+0·01	1898	+0·11
85	+0·04	+0·01	99	+0·11
86	+0·06	+0·02	1900	+0·10
87	+0·05	+0·02	01	+0·09
88	+0·04	+0·02	02	+0·08
89	+0·06	+0·03	03	+0·04
1890	+0·05	+0·01	04	+0·01
91	+0·07	+0·03	05	+0·02
92	+0·08	0·00	06	+0·02
93	+0·99	+0·01	07	0·00
94	+0·10		08	−0·04
95	+0·11		09	−0·01
96	+0·11		1910	−0·02
97	+0·11		11	−0·03

The above was written before I had paid proper attention to Professor E. W. Brown's paper (in *M.N.*, lxxiii. p. 692) on the Moon's longitude. His diagram is easily referred to, and will be seen to be quite consistent with disturbances at Leonid returns. One feature of it specially interests me, viz. that the disturbance near 1800 certainly begins about 1795 and perhaps even earlier, which seems to confirm the conjecture of § 14 of this paper. Professor Brown himself points out the possibility of discontinuities in 1790, 1818, 1863, 1877, and 1897. At first sight the dates 1818 and 1877 present difficulties; but on referring to Professor Brown's diagram it will be seen that 1833 is a good alternative for 1818, while 1877 is merely the end of the disturbance of 1866.

The Perihelion Movement of Mercury.

47. It is well known that the perihelion of Mercury has a secular movement which cannot be explained by the perturbations of known planets, though a hypothetical ring of meteors near the Sun might explain it. But it seems possible that if the Leonids have a sensible mass, they might be responsible for this perturbation, without postulating an unknown swarm. One condition must, however, be fulfilled: the Leonids must find Mercury nearly in the same position at successive returns if their action is to accumulate sufficiently. Now taking a revolution of Mercury as 87·96926 days, we have

$$138 \text{ revolutions} = 33{\cdot}236 \text{ years}.$$

Since 1 revolution = 0·241 years, it is pretty clear that neither 137 nor 139 revolutions would fit the Leonid period with sufficient

exactness. If the Leonids are really responsible, this action on Mercury might provide an accurate determination both of their mass and period. It seems possible that the main mass is concentrated, from what has been noticed about our Moon. But there is certainly a large diffused swarm round it, so that Saturn might find *some* meteors to divert into the SSS within comparatively wide limits. If he hit exactly the main mass, he would doubtless detach more meteors than usual: if his encounter was with the outskirts of the swarm, he might still detach a few. The different circumstances might explain, for instance, the failure to refresh the SSS in A.D. 566, though this may of course be merely a failure of the Chinese records. On the other hand, the planetary perturbations (Earth, Moon, and Mercury) must depend accurately on the passage of the main mass: and hence these perturbations offer the best chance of an accurate determination of the period.

The "Equatorial Acceleration" of Spots.

48. On the view here advocated the spots are due to meteors moving in a retrograde orbit impinging on the Sun's surface. It seems reasonable that they should drive some portion of it (in the neighbourhood of the impact) *backwards*, so that the observed velocity of a spot is not that of the normal surface but smaller. This accords with W. S. Adams's results for the Hα line, which give a greater angular velocity than that observed for spots. On this view the true rotation period of the Sun's surface is that of the Hα line, viz. 24 days.

49. The amount of backdrive will depend on several factors.

(*a*) The velocity of the impinging bodies. This should be approximately the same throughout. It is the perihelion velocity of a swarm moving in a period of 12 years say, with major axis 10, or semimajor axis 5, and grazing the solar surface (distance = ·005 say).

Hence $$V^2 = \mu\left(\frac{2}{\cdot 005} - \frac{1}{5}\right) = 400\mu \text{ say.}$$

For the Earth $v^2 = \mu$.

Hence $V = 20v = 400$ miles per sec. say.

(*b*) The opposite velocity of the Sun's surface, which is greatest at the equator and diminishes with increasing latitude. Hence the backdrive would clearly have least effect at the equator and more and more as we go to higher latitudes. "Equatorial acceleration" is not the proper image: it is polar retardation.

(*c*) The mass of the impinging body or bodies. This is independent of the latitude, and thus we may expect different retardations in the same latitude, as is observed.

(*d*) Any changes in the orbit of the SSS will of course mean changes in V: but these will be slight, since we must suppose the perihelion distance, which provides the predominant element in V^2, to remain the same.

Hence we need not expect serious changes in "backdrive" when the orbit is perturbed by the Leonids every 33 years. But the encounters with Saturn are another matter. They may refresh the SSS with a new set of particles of distinctly different *mass*, so that the backdrive would be essentially altered. Accordingly, Carrington deduced from his observations of 1860–61 a rotation period quite different from that given by modern spots. In between Carrington's observations and ours came the Saturn-Leonid encounter of 1863. The difference of effect is summarised by Mr. and Mrs. Maunder as follows (*M.N.*, lxv. p. 824):—

"(1) Carrington's period does not correspond to a latitude of between 10° and 15°, as he supposed, but is given by the separate spot-groups of latitude 22½°: by the recurrent spots of latitude 20°.

"(2) Carrington's period is not that of the mean of all spots, but is considerably longer than that mean.

. . . . . .

"(6) Spots of short duration tend to give a shorter rotation period than spots of long."

These three of the important conclusions of Mr. and Mrs. Maunder will suffice for our present purpose. That spots of short duration give a shorter rotation is natural: they are presumably due to smaller impinging masses, which produce less backdrive. Now Carrington's period is longer than the modern period, and must therefore have been due to larger impinging masses. This will accord with our hypothesis if we suppose that of the swarm due to each encounter between the Leonids and Saturn the smaller masses fall into the Sun first, and the large ones survive. In Carrington's time only large masses were left in the SSS; but the fresh encounter of Saturn-Leonids in 1863 contributed a stock of fresh material of smaller average mass.

The Formation of Spots.

50. If spots were formed only at perihelion of the SSS, they would originate on the face turned towards the Earth in September and October only. A very casual examination of the records shows that this is not the case. But a little further scrutiny shows that there are distinctly favourable and unfavourable times of year. Miss Bellamy has started a detailed investigation, of which the preliminary results for a few years are available. To select the spots which originate on the face turned towards the Earth she has counted those for which the first record in the Greenwich spot ledgers has a "longitude from central meridian" greater than −70°. Simple counts for the years 1907–11 give the following totals:—

Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	Jun.	July	Aug.	Sept.	Oct.
68	57	72	63	67	75	58	53	51	66	76	67
	197			205			162			209	

There is a distinct minimum about June. This accords with the view that the SSS has retrograde motion; that it is in perihelion in October, and that its bombardment takes place chiefly before perihelion.

[*Note added December* 28.—Several points in the above rapid sketch of a new theory and its possibilities would now be modified: but it is clear that each separate point must be examined with great care before anything like precision could be given to it; and that such an examination will require much time. Hence it has been left just as written, with the exception of the correction of a few obvious slips.]

On the "Kinetic Theory" of Star-clusters. By J. H. Jeans, M.A., F.R.S.

The question has often arisen as to how far it is legitimate to treat the stars of a cluster or of the whole universe as the molecules of a gas, and how far the results of the kinetic theory of gases may be applied to them. The present note attempts to provide an answer, necessarily very limited, to some of the aspects of this problem.

There is no serious difficulty in applying the methods of the kinetic theory to the motion of the most general type of cluster or stream imaginable, but if the conditions are taken to be at all complex, the final formulæ are too complicated to convey any meaning. It is therefore desirable to make as many simplifications as possible.

The first simplification which may be made is to assume all the stars to be of equal mass, m. When numerical computations are made, we shall assume m to be five times the mass of the solar system, or 10^{34} grammes. As regards the density of stars in space, we may accept Lord Kelvin's estimate that there are 10^9 stars within a parallactic distance $0''{\cdot}001$ of the sun. This places the stars at a mean distance apart of $10^{18{\cdot}7}$ cms., or at a parallactic distance of $0''{\cdot}62$, when the stars are assumed arranged in cubical piling. Thus the "number of stars per cubic centimetre," ν, will be $10^{-56{\cdot}1}$. For special star-regions the numerical values may, of course, be widely different.

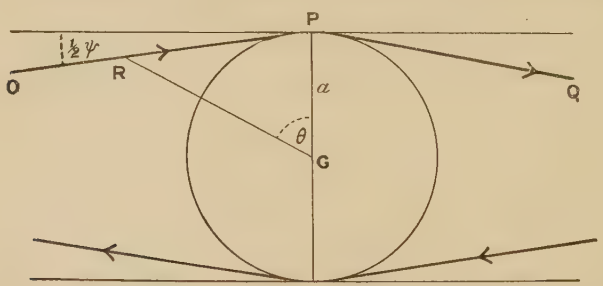
In calculations such as we are now concerned with, the actual velocities of individual stars matter but little; what enters into the formulæ is the relative velocities of pairs of stars. We shall assume pairs of stars to all have the same relative velocity, v; this is much less restrictive than assuming all the stars to have the same velocity, for abnormalities of velocity are largely neutralised when the stars are considered in pairs. We may take $v = 60$ kms. per sec. $= 6 \times 10^6$ as a reasonable value for numerical computations, corresponding to a velocity of about 40 kms. per sec. for a single star.

The last quantity which has to be considered is that which

corresponds to the diameter of the molecule in a gas. A very brief calculation will show that actual collisions between stars are excessively rare; what we need to consider are "encounters" in which each star deflects the other gravitationally from its course. The quantity corresponding to the "diameter" of the gas-molecule will, then, be the diameter of a sphere round each star inside which the gravitational field will be adequate to produce a certain specified effect. And, since effects are more readily produced on slow-moving stars than on rapidly-moving stars, it is clear that the size of this sphere will depend on the assumed velocities of the stars. Apart from this, since the stars are all supposed to have the same mass, the diameter will be the same for each star, say σ.

A similar conception, of course, enters in ordinary gas-theory. When molecules are regarded as point-centres exerting on one another forces which vary as $r^{-\left(\frac{2}{s}+1\right)}$, it is readily shown * that the diameter of the molecule must be supposed proportional to $T^{-\frac{1}{s}}$ where T is the temperature, and therefore to v^{-s} where v is the average velocity. If we take $s=2$, the law coincides with the gravitational law, and σ is seen to vary as v^{-2}. We can, however, easily give a direct calculation in a form suited to stellar problems.

Let O P Q be a slightly curved orbit described by one star about G, the centre of gravity of another equal star and itself.



Let the total deviation in this orbit be ψ, a small angle, and let P G $=a$. If v is the relative velocity, the velocity along O P relative to G is $\frac{1}{2}v$, and this is approximately constant throughout the orbit. When the star is at R, the acceleration along R G is $\frac{\gamma m \cos^2\theta}{4a^2}$, so that the component along P G is $\frac{\gamma m \cos^3\theta}{4a^2}$. The total change of velocity along P G is accordingly

$$\int_{-\infty}^{+\infty} \frac{\gamma m \cos^3\theta}{4a^2} dt = \int_{\frac{\pi}{2}}^{-\frac{\pi}{2}} \frac{\gamma m \cos\theta}{2av} d\theta = \frac{\gamma m}{av},$$

and this must be equal to $v \sin \frac{1}{2}\psi$ or $\frac{1}{2}v\psi$.

* Jeans, *Dynamical Theory of Gases*, p. 257.

Thus to produce a small deflection ψ, α must be equal to $2\gamma m/v^2\psi$, or $\sigma = 4\gamma m/v^2\psi$.

We are left with the consideration of a number of spheres of equal diameter σ moving like the molecules of a gas. Each time a pair of spheres graze one another a deflection equal to ψ will take place; each time the spheres intersect there will be a deflection greater than ψ. A simple process of averaging shows that at each encounter the expectation of deflection is, in point of fact, exactly 2ψ.

To the degree of accuracy to which we are now aspiring, it is immaterial which particular definition of the "free-path" is taken. Taking Maxwell's, it appears that the distance traversed by a star between one encounter and the next is $\frac{1}{\sqrt{2}\pi\nu\sigma^2}$. Thus in a length L traversed by a star, we expect a number of encounters equal to

$$\sqrt{2}\pi\nu\sigma^2 \mathrm{L} \text{ or } \sqrt{2}\pi\nu \mathrm{L}\left(\frac{4\gamma m}{v^2\psi}\right)^2 \quad . \quad . \quad . \quad (1)$$

Calling this number N, and remembering that the expectation of deflection at each encounter is 2ψ, which may be in *any* direction away from the original path, we readily find that the expectation of total deflection in a length L of path is (approximately)

$$2\psi\sqrt{\mathrm{N}} = 8 \times 2^{\frac{1}{4}}\pi^{\frac{1}{4}}\nu^{\frac{1}{4}}\frac{\gamma m}{v^2}\sqrt{\mathrm{L}} = 17\nu^{\frac{1}{4}}\frac{\gamma m}{v^2}\sqrt{\mathrm{L}} \quad . \quad . \quad (2)$$

The result, which will of course be expressed in radians, is independent of ψ, as it ought to be. If we insert the values

$$\gamma = 6{\cdot}66 \times 10^{-8},\ m = 10^{34},\ v = 6 \times 10^6,\ \nu = 10^{-56.1},$$

we obtain for the deflection on a path of L centimetres,

$$\frac{\sqrt{\mathrm{L}}}{v^2} \text{ radians} = 2{\cdot}8 \times 10^{-14}\sqrt{\mathrm{L}} \text{ radians} = 1{\cdot}6 \times 10^{-12}\sqrt{\mathrm{L}} \text{ degrees.}$$

Thus a deflection of 1° may only be expected after a distance of 4×10^{23} cms., which with a velocity of 4×10^6 cms. per second requires 10^{17} secs. or 3200 million years to describe. A deflection of 1′ may be expected in $\frac{1}{3600}$th of this time, say roughly a million years.

The formula just used has, of course, no application to the occurrence of single violent encounters; to study these we go back to formula (1), which shows that an encounter producing a deflection at least equal to ψ may be expected after a path equal to

$$\frac{1}{2^{\frac{1}{2}}\pi\nu}\left(\frac{v^2}{4\gamma m}\right)^2\psi^2 \text{ cms.} = 8{\cdot}5 \times 10^{28}\psi^2 \text{ cms.}$$

For a deflection of 5° (= ·087 radians), the required length of path is found to be 6×10^{26} cms., which with a velocity of 4×10^6 cms. requires a time of 5×10^{12} years; for a deflection of 2° the time will be 16 per cent. of the above, say 8×10^{11} years.

To illustrate the use which can be made of the formula, let us take a definite instance of a star-stream in which the stars all

start with equal and parallel velocities of 4×10^6 cms. per sec. Let us suppose that a star is still considered to belong to the main stream as long as its direction of motion makes an angle not greater than 2° with the main stream. After 100 million years, the stream will have lost only one in 8000 of its original members, and the remainder will make angles with the main stream of which the average amount is only 10′. After 3200 million years, the loss will be one in 250, and the average angle will be 1°. After 80,000 million years, one-tenth of the original members have been lost by violent encounters, but the average angle of the remainder is 5°. Thus the stream has been mainly dissolved, not by collisions or violent encounters, but by gradual scattering.

From these figures it will be obvious that there can be no question of a universe similar to ours coming to a final steady state such as we are familiar with in the theory of gases. The time which Maxwell calls the "time of relaxation" is of the order of 10^{14} years, and even after the lapse of this time the universe would not be in a steady state but only $1 - e$th of the way on towards such a state. For closely packed star-clusters the result might, of course, be very different.

Distribution of Sunspots in Heliographic Latitude, 1874–1913. By E. Walter Maunder. (Plates 2–4.)

In the years 1903 and 1904 four papers were communicated to this Society on the general subject of the distribution of sunspots in heliographic latitude. Of these, one was communicated by the Astronomer Royal, and appeared in the *Monthly Notices*, vol. lxiii. p. 452; the second was by Dr. W. J. S. Lockyer, and appeared in the Appendix to vol. lxiv. p. [5]. In it the author drew from the first-named paper certain inferences, which called forth the third and fourth papers. These were by the Rev. A. L. Cortie and myself respectively, and appeared in vol. lxiv.; my own paper appearing on p. 747. My intention in it was to supplement the Astronomer Royal's paper, and to correct what I thought inadmissible in Dr. Lockyer's inferences.

My paper, which was dated 1904 June 9, was illustrated by eight diagrams, of which the last was the most important. This showed, rotation by rotation, for the period 1877–1902, the distribution of spot centres in heliographic latitude, and it formed Plate 16 of vol. lxiv. of the *Monthly Notices*.

As the statistics for a complete further cycle of sunspot activity are now available, it seems to me that it might be desirable to bring this diagram up to date; the more so that the Greenwich sunspot record has been supplemented by the publication in 1907 of the Sunspot Ledgers for the years 1874–1885, based not only upon the measures of photographs obtained at Greenwich, but also upon photographs taken at Harvard College, at Melbourne, in India, and in Mauritius.

Fig. 1 (Plate 2), therefore, corresponds to the diagram published in 1904, but represents the observations of nearly forty years instead of nearly twenty-six. As before, the intention has been to show for each synodic rotation period of the Sun, the distribution in heliographic latitude of the centres of the individual spot-groups. Whenever, during a given rotation, the centre of a group of spots has fallen in a particular degree of latitude on one or more days during the rotation, a line has been drawn across the space corresponding to that degree. In assigning a spot centre to any particular degree, as say 7°, all latitudes from 6°·5 to 7°·4 inclusive have been reckoned at 7°; and similarly in other cases. The diagram represents distribution in latitude but takes no account of area.

An examination of the present diagram, as of that published in 1904, brings out Spoerer's Law with remarkable clearness. There is but a single movement shown in either hemisphere, the trend of which is downward, from one minimum to the next minimum; so that the general form of the latitude lines during each of the three complete cycles is that of a hollow wedge.

The diagram represents very nearly forty years, and shows in whole or in part five successive movements in latitude. The end of the first movement is shown, the whole of the second, third, and fourth, and the beginning of the fifth.

Many inquiries have been made of late in order to ascertain whether there are other periods in sunspot activity than that one which accomplishes its course on the average in 11·1 years. It is claimed that there is a period of about 33·3 years in length; approximately of the Leonid meteors. The 11-year period is a submultiple of this, that is to say one-third, and it is claimed that other submultiples of the great Leonid period, namely one-fourth, one-sixth, one-seventh, and one-ninth, are also veritable periods of sunspot activity; 4·79 years—one-seventh of the Leonid period—being specially persistent and distinct.

It is sufficient to say that fig. 1 shows clearly that the sunspots in the last forty years—the forty years for which we have the most complete and continuous observations—have accomplished not merely one, but three complete movements in latitude; and they have accomplished no more than three.

Fig. 2 (Plate 3) gives, year by year, from 1854 to 1912,* the mean daily area of sunspots, and the mean distance from the equator of the entire spotted area; and it shows that the areas of the spots have had exactly the same periodicity as their latitudes. Six times in the fifty-nine years we have had epochs of almost complete solar quiescence. Five times, therefore, there has been a complete progression from a state of minimum activity up to maximum and down to minimum again. This complete progress

* Spoerer's figures have been used in this diagram up to the end of 1877; the Greenwich figures afterwards. The relative numbers given by Spoerer do not correspond exactly to the Greenwich areas, but the difference of scale does not affect the present question.

has taken place five times and no more in the fifty-nine years; the progressive change in latitude and the progressive change in area have run their courses together.

If we take any particular zone of latitude in either hemisphere, whatever be the breadth which we assign to that zone, the relation between the areas of the spots and their distribution in heliographic latitude is so close that each separate zone gives us the same phenomenon as the entirety of the whole. That is to say, each zone gives us six minima, involving five complete progressions from minimum up to maximum, and down to minimum again; five, and no more and no less than five. The only exceptions that we can find to this law are in the highest latitudes, where spot-groups appear to be merely sporadic, for at all times they are here very small, short-lived, and infrequent.

In the early days of a cycle, when the Sun's activity is rapidly increasing, the zones chiefly affected are those between 33° and 23°; it is here that the nascent activity develops its first energies. But these are zones which during the greater part of the 11-year period are absolutely barren; nevertheless, they are clearly connected in the most intimate fashion with the activity of the zones nearer the equator. It is in them that the reviving activity begins; it is from these zones that it spreads downwards to the regions of greatest disturbance.

The equatorial zone, though in a less marked degree, bears witness to exactly the same phenomenon. Spots on the equator are at no time very numerous, but we only lose them entirely once in every 11-year period. This happens just at the very epoch when sunspot activity in general is most markedly on the increase.

In short, no matter what zone we take, we find that once and once only in each 11-year period there is, for that particular zone, a prolonged time of absolute quiescence. The time of quiesence may amount in the higher zones to six or seven years or even longer; in the lower to two years or perhaps a little less. Of course, if we confine our consideration to a very narrow zone, we shall find that this may display a somewhat checkered activity; it may not show spots in every rotation, even when the zone is most active. But it is only once in the 11-year period that a zone will remain barren for so long as twenty up to one hundred consecutive rotations.

But this "checkered activity" of narrow zones, which is shown by fig. 1, is much more the effect of the conditions which I had to impose upon myself in preparing the diagram, than of the real state of the case. If I had represented in the diagram the breadth of each group as well as the position of its geometrical centre, the majority of the lines drawn for the successive rotations would have been continuous; few would have appeared interrupted. It will, of course, be understood that the preparation of a diagram of the kind demands a considerable amount of labour, which would be much increased if the breadth of each group was indicated as well as its position. Nevertheless, I think this

Fig. 1.—Distribution of Spot-centres in Heliographic Latitude.

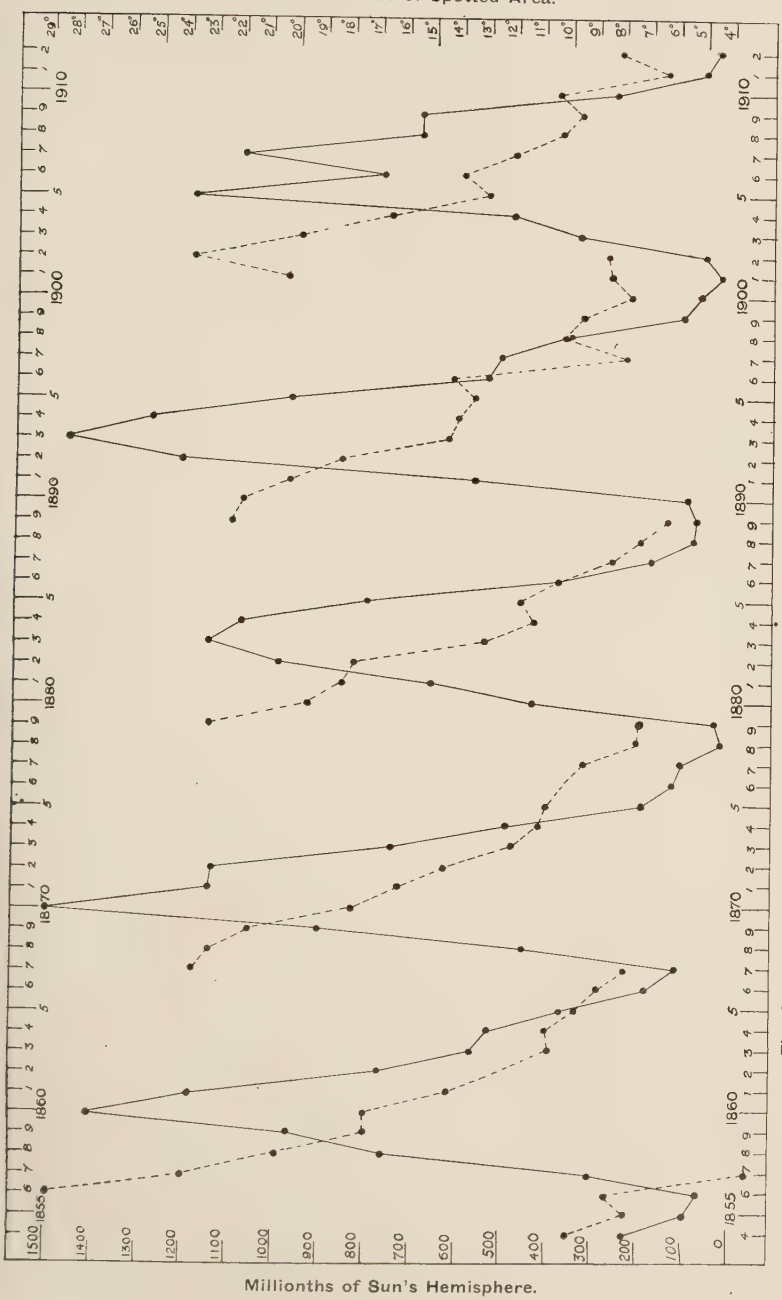


Fig. 2.—Mean Latitude and Mean Spotted Area, 1854-1912.

modification would be well worth attempting, and I hope in the future to be able to carry it out. Its only effect, however, would be to render more conspicuous the fact that the period of 11·1 years is the only one in evidence.

The two hemispheres, the northern and southern, have the same period, but with a slight difference in the phase. During the forty years covered by fig. 1; the northern hemisphere has been a little in advance of the southern in all its phases, but has, on the whole, shown somewhat less activity. Thus, so long as the Sun's activity is on the upgrade, the northern hemisphere has shown more activity than the southern; the southern has come into action a little later, but with greater emphasis. It does not appear that this slight difference in the behaviour of the two hemispheres characterises them in all cycles; but, however this may be, the general accord between the two hemispheres is a further indication that the exciting cause of solar-spot activity is essentially one, and that it affects the Sun as a whole. The mean length of the period is the same for each hemisphere, and for each several zone in each hemisphere.

Of all the zones on the Sun, the most important, from the point of view of the development of the sunspot cycle, are those in which the Sun's reviving energy is initiated; namely from 33° to 23°. But these are the zones with the longest durations of quiescence at their minima. Of all the alleged periods of sunspot activity, other than that of 11 years, the one to which most prominence has been given is that of 4·79 years—practically three-sevenths of the 11-year period. Since the zones of nascent activity remain absolutely barren for more than five years at a time, it follows that, for them, the 4·79 period must always be non-existent for at least three out of every seven oscillations. Or to put it another way, three and often four maxima out of every seven are zero. I think the fact is sufficient to show that as a sunspot period 4·79 years is merely imaginary. Where there is no activity at all, where the curve never rises above the base line by so much as a fraction, how can it be determined where the maximum falls and where the minimum?

The preceding considerations refer to the zones of nascent activity. But the Sun, as a whole, shows, though less distinctly, just the same indifference to the 4·79-year period. Fig. 3 (Plate 4) exhibits for each year, from 1826 to the present time, the percentage of days of observation upon which the Sun appeared to be free from all spots, the figures prior to 1874 being derived from Schwabe's results; after 1874 from the Greenwich observations. It will be noted that, from whatsoever date we begin to reckon the working of the 4·79 period, one or more maxima will fall at times when the Sun as a whole had almost ceased to develop spots.

Yet the period of 11·1 years is not the only one affecting sunspot observations. There is another which of necessity affects all our statistics: it is the synodic period of the Sun's rotation. However large a group of spots may be, whatever its latitude, it can

only be observed when it is in the hemisphere of the Sun which is turned towards the Earth. But the Sun's activity is at all times developed very irregularly in longitude; it is not uncommon for a single meridian to be markedly predominant in activity for three or four rotations in succession; or, with occasional interruptions, for a longer period—thus giving rise to a well-marked periodicity of about 27 or 28 days. This periodicity, however, becomes blurred when one meridian ceases to be predominant, and another takes its place. Since a few of the greatest sunspot disturbances completely dwarf all the smaller ones as to area covered, number of individual spots formed, and length of time for which the disturbances last, they are able, by their numerical supremacy, to introduce into sunspot statistics those complications from which purely statistical methods have derived these fictitious periods.

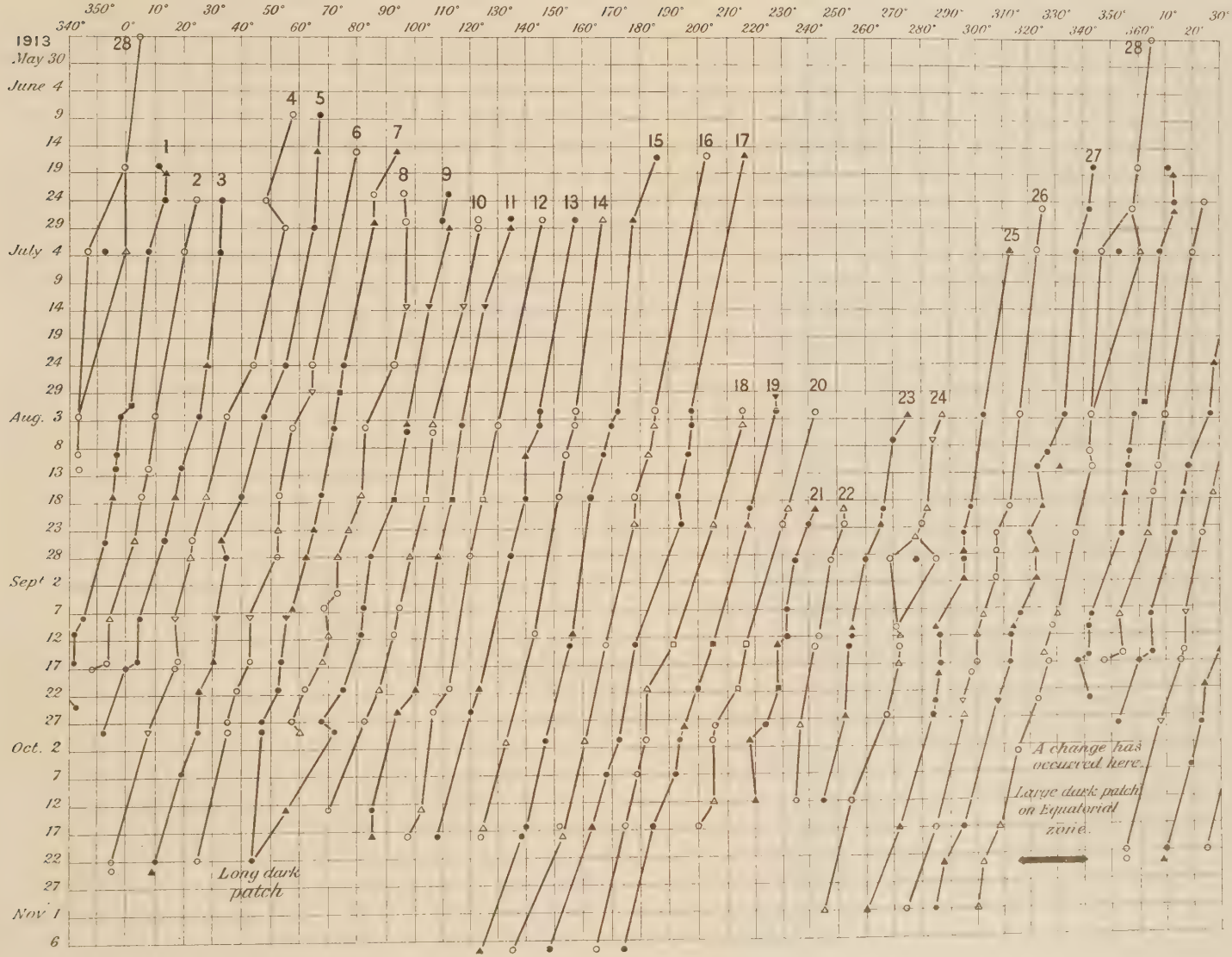
8 *Maze Hill, Greenwich Park:*
1913 *December* 5.

The Equatorial Current of Jupiter in 1913. By Rev. T. E. R. Phillips, M.A. (Plate 5.)

In the late summer of 1912 the north equatorial belt of Jupiter, which had been passing through a time of quiescence, began to exhibit a revival of activity. On 1912 August 23 Mr. J. J. Schafer of Port Byron, Illinois, noted a dark and conspicuous spot on the belt, which was quickly followed by the appearance of others, and the disturbance spread rapidly. Unfortunately, the planet was badly placed for observation, and no reliable value of the rotation period seems to have been obtained.

On the reappearance of Jupiter in the spring of 1913 it was found that the north equatorial belt was broad and strong, with a series of very dark protuberances along its south edge and light bays between them. These protuberances still further developed as the apparition advanced, and a number of curved wisps were seen to extend from them like festoons, which completely enclosed the light areas. Finally, the south portion of the zone became very much shaded, and as many of the white spots simultaneously brightened, the planet for some time presented the appearance of a number of brilliant egg-shaped areas ranged with remarkable regularity round the north part of the equatorial zone. Towards the close of the observations it was seen that the north part of the north equatorial belt had faded considerably, and the disturbance showed signs of declining activity.

At Ashtead the weather and seeing conditions during the late summer and autumn proved very favourable, so that despite the low position of the planet in the sky a large number of transits (nearly 350) of the dark and light areas were obtained during the apparition. These showed that the main spots were 28 in number (14 dark and 14 light), and that they were all drifting at



(System I.)

North Equatorial Spots in 1913. Longitudes of spots from observations made at Ashtead.

	Observer
● Dark spot (▲ *Transit estimated*)	*T.E.R. Phillips.*
○ Light spot (△ *Transit estimated*)	*Do.*
■ Dark spot	*H.F. Acocks.*
□ Light spot	*Do.*
▼ Dark spot	*Miss E.K. Church.*
▽ Light spot	*Do.*

an unusually rapid rate. Details of the spots and their rotation periods are given in the accompanying table, from which it will be seen that the mean period derived from the whole series is

$$9^{h}\ 50^{m}\ 12^{s}\cdot2.$$

Table of North Equatorial Spots in 1913.

No.	Character of Spot. D = dark. W = white.	Approximate Longitude (System I) on 1913 Aug. 1.	No. of Days covered by the Observations.	No. of Observations.	Diminution of Longitude per Month (30 days).	Rotation Period from whole Series of Observations.	Remarks.
		°			°	h m s	
1	D	0	99	16	10·6	9 50 15·7	United with 3 to form one dark mass at end of September.
2	W	11	85	9	11·3	14·8	Obliterated by 1 and 3 at end of September.
3	D	25	97	11	12·4	13·3	Very dark.
4	W	36	136	14	15·0	9·8	
5	D	49	136	15	13·7	11·6	
6	W	61	129	13	13·3	12·1	
7	D	72	129	16	12·3	13·5	United with 9 to form one mass in October.
8	W	88	99	15	12·1	13·7	Died out early in October.
9	D	99	113	14	14·3	10·8	
10	W	110	108	12	14·7	10·2	
11	D	120	113	10	14·3	10·8	
12	W	131	129	9	12·6	13·1	
13	D	144	129	10	11·9	14·0	Very dark and prominent.
14	W	158	129	10	11·9	14·0	
15	D	170	145	12	11·6	14·4	
16	W	185	145	11	13·0	12·5	
17	D	199	145	11	14·3	10·8	
18	W	217	99	9	17·0	7·1	
19	D	228	101	11	16·6	7·7	
20	W	242	76	9	17·0	7·1	
21	D	251	53	10	14·1	11·0	Very dark and prominent.
22	W	260	53	7	12·5	13·2	Intensely brilliant.
23	D	271	71	10	11·4	14·7	
24	W	287	91	13	12·5	13·2	
25	D	302	121	15	12·1	13·7	Very dark.
26	W	318	128	15	12·4	13·3	
27	D	332	137	18	12·7	12·9	
28	W	346	160	17	12·4	13·3	
Means	...	...	113	12	13·2	9 50 12·2	

The motion, however, was not uniform, and after the middle of August the more slowly moving spots became accelerated, with the result that the mean period from August 15 to the beginning of November was still further reduced to

$$9^h\ 50^m\ 10^s{\cdot}1.$$

To enable the identifications and rotation periods to be checked, the individual observations are presented in chart form. I regret that sufficient observations of spots in the south part of the current were not obtained at Ashtead in 1913 to enable trustworthy results to be deduced; but if we assume that the velocity of the current as a whole was fairly represented by the north equatorial spots,* it is interesting to compare the present velocity with that found in former years. From a consideration of previous determinations (see Mr. A. S. Williams' discussions in *M.N.*, vol. lviii. p. 14; vol. lxxi. p. 145; *Zenographical Fragments*, vol. ii. p. 100) it appears that the rotation period steadily increased from about $9^h\ 50^m$ in 1879 to about $9^h\ 50^m\ 30^s$ in 1889. During the next 25 years or so, though showing a number of minor fluctuations, it remained fairly constant, with some indication of a slight diminution for a few years about 1900. I have no reliable results for the equatorial current in 1912, but during the present year the acceleration has been so marked that the rate now seems to be practically identical with that of 30 years ago. It is important to determine when the planet reappears whether this acceleration is being maintained, while future years will show whether such changes are regularly periodic or otherwise.

Ashtead:
1913 *December* 11.

The Spectra of Wolf-Rayet Stars. By J. W. Nicholson, M.A., D.Sc., Professor of Mathematics in the University of London.

In taking up the examination of these spectra, with the object of arranging them in some mathematical manner, we must consider, in the first place, what phenomena they are likely to present, in view of current theories of their constitution.

In recent years many astrophysicists have tended to become convinced that the Wolf-Rayet stars are probably the earliest type with which we are acquainted, or, in fact, that they are one of the earliest products of some form of evolution which has taken place in primitive nebulæ. The evidence leading towards this conclusion, which we shall attempt to justify in the present paper from a new point of view, is derived from a variety of different features in the spectra of these stars. In view of an interpretation

* From the results of various observers in 18 years between 1886–7 and 1909–10, I find that the mean rotation period of the north part of the current was in these years $3^s{\cdot}4$ longer than that of the south part. This difference, however, was not constant. In 1886–7 the former was nearly 18 seconds greater, and in 1888 about 4 seconds less than the latter.

of an important part of the nebular spectrum, already given,* we must apparently expect to find evidence in these stars of a substance, or of a series of substances, chemically distinct from the theoretical element nebulium, but at the same time constituting simple modifications of this element. These elements may have no terrestrial existence, or, on the other hand, they may be the proto-elements of Lockyer, or more primary forms of the terrestrial elements.

Campbell,† to whom we are mainly indebted for a knowledge of the characteristic spectrum of a Wolf-Rayet star, has given a comparison of this spectrum with that of Nova Aurigæ at a pre-nebular stage, and with certain chromospheric lines and dark lines in the spectra of Orion stars and of β Lyræ. The comparisons with the dark stellar lines and with the lines of Nova Aurigæ appear to be fairly convincing, but those with chromospheric lines, which identify the origins of several Wolf-Rayet lines with the terrestrial elements iron and magnesium, are of a different character, so that Campbell himself lays little stress upon them. The coincidences are not close, and the relative intensities are in no accord with those observed in the solar chromosphere. In a spectrum like that of iron, with its multitude of lines, some coincidences must take place with any comparison spectrum.

Apparently it is necessary at present to regard almost the entire Wolf-Rayet spectrum as of unknown origin. The only ordinary elements which are certainly present are hydrogen and helium, and the latter is not well represented, unless Pickering's series and the line $\lambda 4688$ are ascribed to helium instead of hydrogen. In a recent note‡ it has been shown that certain other lines form a well-defined series of the form

$$\lambda = \lambda_0 \frac{(m \pm \frac{1}{3})^2}{(m \pm \frac{1}{3})^2 - 4},$$

where λ_0 has the value for Balmer's hydrogen formula. These lines were accordingly ascribed to hydrogen by analogy with the Pickering lines. More recently, Bohr§ has suggested that the Pickering lines are due to helium, and that this new series is due to lithium—in the former case with a single positive charge, and in the latter with two positive charges. As the lines are strong in these stars, such a view appears to imply that the Wolf-Rayet stars are, in the main, possessed of a large positive charge of electricity, and are chemically made up from the elements of low atomic weight. It will be seen later that such a view, if it should prove ultimately to be correct, is not at variance with the considerations in this paper, for the substances referred to later as "evolution products" may be the ordinary chemical elements when they are sufficiently stable, showing traces, in the spectra of the stars, of less stable intermediate products of the transition from nebular elements.

The object of this paper is to put forward an account of what

* *Monthly Notices*, Nov. 1911. † *Astronomy and Astrophysics*, vol. xiii.
‡ *Monthly Notices*, Feb. 1913. § *Phil. Mag.*, July and Sept. 1913.

the writer believes to be the nature of the Wolf-Rayet spectrum. The full discussion of the phenomena presented by the spectrum, which is by no means identical in all these stars, would require much space, so that the present paper is in the nature of an introductory treatment only, whose material data are limited to the typical *arerage* spectra of these stars, as given by Campbell in his table. More recently,* Dr. Max Wolf has made an exhaustive set of observations of four Wolf-Rayet stars, but the present paper, dealing with the spectra in a general way, will not take account of this work. There is reason to believe that the novæ possess a Wolf-Rayet stage, and can, at some time in their history, be regarded as following out the evolution from nebulæ in a retrograde direction. This question also cannot be discussed profitably except in connection with Wolf's results.

Some remarkable series relations exist among the Wolf-Rayet lines, and the constants in these series are related to the vibration-constants of the atom of nebulium. For the present, no theory of these relations is suggested, and the formulæ are regarded as empirical. But, even as empirical formulæ, they point very strongly to the conclusion that the Wolf-Rayet stars are a very early evolution product of the nebulæ, for they link up the spectra very completely with at least a part of the nebular spectrum.

Campbell's table of the Wolf-Rayet spectrum is given below, together with his comparison with that of Nova Aurigæ. The other comparisons are not included. The symbol (1) denotes a strong line, and (2) a very strong one.

Wolf-Rayet.	N. Aurigæ.	Wolf-Rayet.	N. Aurigæ.
5848		4509 (2)	
5813 (2)		4504 (1)	
5693 (2)		4493	4490
5593 (1)		4480 (1)	4481 (1)
5472 (1)		4473 (1)	4473
5284	5285	4466 (1)	
5250		4457	
5131		4442 (2)	4445 (1)
5020	5016 (2)	4416	4418 (1)
4940		4389 (1)	
4787		4369	
4652 (2)		4334	
4636 (1)		4318	4316 (1)
4626 (1)	4629 (1)	4273	
4615		4260	4262 (1)
4596		4228	4228
4555	4556 (1)	4063 (1)	4067 (1)
4517 (1)		4023 (1)	

* *Sitzungs. der Heidelberger Akad.*, Jahrgang 1913.

The table contains thirty-six lines. Those of the Balmer and Pickering series, together with λ 4688 and D_3, are omitted. D_3 is ordinarily present, but not strongly, and λ 4688 is usually strong. With these exceptions, all the lines found by Campbell are included. Wolf's observations give many more, but usually weaker lines, of less importance in a preliminary discussion, but decisive later.

When these wave-lengths are examined, it is found that several pairs are almost exactly in the ratio 5 : 4. Thus

$$\frac{5285}{4228} = 1{\cdot}2500, \qquad \frac{5693}{4555} = 1{\cdot}2499,$$

$$\frac{5813}{4652} = 1{\cdot}2495, \qquad \frac{5593}{4473} = 1{\cdot}2503.$$

Although these so-called "lines" are usually the centres of rather broad bands, they are the mean, in each case, of a large number of observations, which do not differ among themselves by more than 3 or 4 tenth-metres. It is probable that the estimates of wave-length, in these cases at least, are accurate to the last figure. A change of a fraction of a tenth-metre will in each case make the ratio exact. The closeness of these values to 5/4 is very striking, and it seems improbable, at the outset, that it is due to chance in every case. With some of the lines above we are not concerned in this paper, but it is sufficient to say now that further investigation shows these ratios to be more than chance coincidences, and that the whole spectrum can be elucidated with this clue.

Consider now the formula

$$\lambda = \lambda_0 \frac{m^2}{m^2 - 1},$$

which is similar to Balmer's formula, when m takes integral values.

When $m = 2$ and $m = 4$ we obtain

$$\lambda_1 = \frac{4}{3}\lambda_0, \qquad \lambda_2 = \frac{16}{15}\lambda_0,$$

or $\lambda_1/\lambda_2 = 5/4$. This suggests the presence of series of Balmer's type in the spectra of Wolf-Rayet stars, though, as the stars are faint and rather inconspicuous, perhaps only one or two members of each series should be expected. Confining attention for the present to the lines λ 5285 and λ 4228, for which the ratio 5/4 is exact, and writing

$$5285 = \lambda_0 \frac{2^2}{2^2 - 1}, \qquad 4228 = \lambda_0 \frac{4^2}{4^2 - 1},$$

we obtain $\lambda_0 = 3963{\cdot}5$, and thence

$$\lambda_0 \frac{3^2}{3^2 - 1} = 4458{\cdot}9.$$

There is a Wolf-Rayet line at λ 4457,—perhaps not a very accurate measurement, as the line is faint and not of regular

occurrence in Campbell's observations. It can be regarded as within the experimental degree of accuracy. Thus a series

$$\lambda = 3963{\cdot}5\frac{m^2}{m^2-1},$$

with members

$m=2,$	$m=3,$	$m=4,$
$\lambda=5285,$	4457,	4228,

may exist in the spectrum. None of these three lines is of great intensity. The next line of the series would be at $\lambda=4128{\cdot}6$, and is not in Campbell's list. As already stated, this is the only list dealt with in the present paper.

Another series of this type also appears to exist, with the same degree of probability. For the first four members of the series

$$\lambda = 4104\frac{m^2}{m^2-1}$$

are given by

$m=2,$	$m=3,$	$m=4,$	$m=5.$
$\lambda=5472,$	4617,	4377,	4275.

The first is a strong Wolf-Rayet line, and the second is of usual occurrence. The third has not been found by Campbell, who has, however, detected the fourth. Three of the four lines are therefore in the spectrum, and there is a strong case for further examination.

Relation to Nebulium.

The limits of the preceding series are 4104 and 3963·5. Although the form of the series suggests a relation to hydrogen, by analogy with Balmer's formula, investigation does not appear to lead to any simple relation between either of these limits and the limit 3647 of that formula. It seems unlikely, therefore, even at this stage, that the chemical origin of these series, if they exist as such, can be hydrogen. In a similar manner, it does not seem to be helium.

In proceeding further with the discussion of the existence of series relations between the Wolf-Rayet lines, the first question which arises is that of the possible existence of more than one series with the same limit. On the supposition that the Wolf-Rayet stars are of a simple chemical nature, a type of series to be expected is

$$\lambda = \lambda_0\frac{m^2}{m^2-\alpha^2},$$

for the Rydberg constant μ is always unity for hydrogen, or one-half if the Pickering series is really due to hydrogen. Moreover, the only helium line to which any certainty attaches is D_3, belonging to the diffuse series for which this type of formula is very accurate. Other considerations, which need not be specified here, lead to the same conclusion.

But whatever the form adopted, if there are two series with the same limits as before, the ratio of the lines of the two series, corresponding to a given value of m, will be that of the limits, or

$$\frac{4104}{3963{\cdot}5} = 1{\cdot}0354.$$

Lines in the Wolf-Rayet spectrum which have this ratio are λ 4472 and λ 4318. Thus

$$\frac{4472}{4318} = 1{\cdot}0356,$$

and the order of accuracy is sufficient.

The magnitude of the wave-lengths indicates that they probably correspond to $m = 2$, and, taking λ 4472 as derived from a series of limit 4104, of the form mentioned above, we find

$$a^2 = 0{\cdot}3286.$$

This indicates a series

$$\lambda = 4104\frac{m^2}{m^2 - 0{\cdot}3286},$$

whose members are

$m = 2,$	$m = 3,$	$m = 4.$
$\lambda = 4472,$	4260,	4190.

The second is also a comparatively weak Wolf-Rayet line, so that the existence of the series is rendered likely. The accompanying series with the other limit,

$$\lambda = 3963{\cdot}5\frac{m^2}{m^2 - 0{\cdot}3286}$$

gives

$m = 2$	$m = 3,$
$\lambda = 4318$	4114,

and the first is again a line in Campbell's list. Even if that list alone is considered, there is a very strong case already for the existence of this group of series. The two series of limit 3963·5 are weaker than those of limit 4104.

It must be pointed out that the line λ 4472 is generally ascribed to helium, and is one of the very limited number which Wolf-Rayet stars possess in common with nebulæ. But the striking absence of all other helium lines except D_3 and λ 4389 (whose origin from helium may in the latter case be disputed), and the weakness of D_3 when it occurs, is an argument against this view. Moreover, there is no doubt that λ 4472 is composite in these stars, with probably even three components.

But, as already explained, the object of this paper is not so much to discuss individual lines as to supply a volume of cumulative evidence that this series representation of the Wolf-Rayet spectrum is real. The treatment is therefore strictly general for the present.

We proceed accordingly to an immediate examination of the

relation between the four series whose existence is suggested, viz.,

$$\lambda = (3963{\cdot}5, \quad 4104)\frac{m^2}{m^2 - 1},$$

$$\lambda = (3963{\cdot}5, \quad 4104)\frac{m^2}{m^2 - 0{\cdot}3286},$$

which already include, as their leading members, nine Wolf-Rayet lines from Campbell's list.

The constants in the series are related to those of the atom of nebulium, introduced in an earlier paper.* In the notation of that paper, let ω denote the angular velocity of a ring of electrons in steady motion around the positive nucleus of the atom, and let the frequency of the vibration of the atom about its steady motion be $q/2\pi$. Then it is known that for an atom with a given number of electrons and a nucleus of given strength, q/ω can take a series of numerical values which may be calculated. A small correction in some cases for the mobility of the nucleus is neglected, for its effect is not appreciable in the results below. Then in particular, for the neutral atom of nebulium, we find for the three main vibrations, the values,†

$$q^2/\omega^2 = 1, \qquad 16/(15 - 2\sqrt{2}), \qquad 8/(8 + \sqrt{2}),$$

or

$$q^2/\omega^2 = 1, \qquad 1{\cdot}3145, \qquad 0{\cdot}8496.$$

Now $0{\cdot}3286 = \frac{1}{4}(1{\cdot}3145)$, suggesting that the constant 0·3286 found in the second set of series bears an important relation to the atom of nebulium. If the Wolf-Rayet stars are indeed derived from nebulæ by some evolutionary process, this result is very significant. The subsequent investigation in this paper tends to confirm this supposition in other directions, so that the discussion of the reality or otherwise of this relation may be omitted. But it is at once suggested that other series may be expected of the type

$$\lambda = (3963{\cdot}5, \quad 4104)\frac{m^2}{m^2 - q^2/4\omega^2},$$

where q^2/ω^2 can have *all* the values pertaining to the neutral atom of nebulium. At the same time, the further series may be expected to be much weaker if their intensity bears any relation to the strength of the corresponding vibration in the simple nebulium atom, or, in other words, to the degree of simplicity of the vibration concerned.

We can obtain confirmation of the probable existence of such series at once. For example, one of them has a limit 4104, and a value of $q^2/\omega^2 = 0{\cdot}8496$, giving

$$\lambda = 4104\frac{m^2}{m^2 - 0{\cdot}2124}$$

as the formula for the series. The lines are:—

$m = 2$	$m = 3$.
$\lambda = 4334$	4203.

* *Monthly Notices*, Nov. 1911. † *Loc. cit.*

The first is one of Campbell's lines, and weak, as would be expected. There is also a weak line at λ 420 (Campbell), but this may be more probably a line of Pickering's series.

The Third Set of Series.

Selecting the lines corresponding to $m = 2$, in the three series of limit 4104, we obtain

$$\lambda\lambda\ 4334,\quad 4472,\quad 5472.$$

The question arises: Are there other series which differ from these only in the value of the limit? For the existence of two *sets* of series converging to different limits, but otherwise identical in form, has now been rendered probable. The existence of others can be tested by examining the spectrum for another trio of lines in the same mutual ratios as the above, or, in other words, by finding wave-lengths $(\lambda_1\ \lambda_2\ \lambda_3)$ satisfying

$$\frac{\lambda_1}{4334} = \frac{\lambda_2}{4472} = \frac{\lambda_3}{5472}.$$

If these are found, we may expect that they correspond to $m = 2$, in another set of series differing only in the value of the limit.

Such lines can be found. For, taking

$$\lambda_1 = 4492, \qquad \lambda_2 = 4636,$$

we have

$$\frac{\lambda_1}{4334} = 1{\cdot}0365, \qquad \frac{\lambda_2}{4472} = 1{\cdot}0366,$$

the ratios coinciding to 1 part in 10,000. The corresponding value of λ_γ would be about 5700, where weak lines have not been looked for. We can test the existence of the third series later, however, by its members of smaller wave-length.

The above ratios suggest that λ 4636 and λ 4492 are the lines corresponding to $m = 2$ in series

$$\lambda = \lambda_0 \frac{m^2}{m^2 - 0{\cdot}3286}, \qquad \lambda = \lambda_0 \frac{m^2}{m^2 - 0{\cdot}2124},$$

which should be accompanied by a third series

$$\lambda = \lambda_0 \frac{m^2}{m^2 - 1},$$

for the value $m = 3$ at least, and perhaps for greater values of m. Calculation gives at once $\lambda_0 = 4254$ as the necessary limit.

Examining the series in detail, we find that the members of

$$\lambda = 4254 \frac{m^2}{m^2 - 0{\cdot}3286},$$

which, by analogy with previous work, should be a strong series, are

$m = 2$	$m = 3$	$m = 4$.
$\lambda = 4636$	4415	4343.

Not only the first line is found in these stars, *but also the second*. The third, if present, would be masked by Hγ.

The series

$$\lambda = 4254\frac{m^2}{m^2 - 0{\cdot}2124}$$

gives only its first member, as would be expected, for its value of q/ω corresponds to the very weak transverse vibration of Class I. in the simple nebulium atom.

The third series

$$\lambda = 4254\frac{m^2}{m^2 - 1}$$

gives for $m = 3$, $\lambda = 4787$, which is a Wolf-Rayet line. This series also therefore seems to exist, although its first member is towards the infra-red end of the spectrum. This set of series, finally, is as well established as those given earlier.

The Constant Difference Law.

We now have the series

$$\lambda = (3963{\cdot}5,\quad 4104,\quad 4254)\frac{m^2}{m^2 - q^2/4\omega^2},$$

where q/ω has a purely theoretical set of constant values, and the only constants which are still arbitrary are the limits of the series. This arrangement has so far included the following fifteen Wolf-Rayet lines :—

5472,	5285,	4787,	4636,	4615,
4492,	4472,	4457,	4415,	4334,
4318,	4273,	4260,	4228,	4203,

where the last line, however, probably has a better interpretation in terms of the Pickering series. Since of these lines only 4472 has ever received any suggested interpretation (in terms of helium), the cumulative evidence in favour of these simple series is very strong. About half the lines of the spectrum are contained in the list.

The formulæ are further strengthened by a remarkable relation between the three limits,

$$\lambda_0 = 3963{\cdot}5,\quad 4104,\quad 4254,$$

for their wave-numbers are

$$\frac{10^8}{\lambda_0} = 25227,\quad 24366{\cdot}5,\quad 23506,$$

and their successive differences are 860·5, 860·5, *and are identical.* This almost compels a belief in the physical reality of the formulæ. The whole set of series can now be grouped into the form

$$\lambda = \lambda_0\frac{m^2}{m^2 - q^2/4\omega^2}$$

where $\lambda_0^{-1} = A + Bn$, n taking integral values, and A and B being definite constants at present still arbitrary. q/ω has a set of

theoretically calculable values. Thus the whole fifteen lines depend only on two arbitrary constants, for the values of q/ω are independent of the actual wave-lengths of the nebular lines which have been ascribed to nebulium in a previous paper.

Further Examination.

Hitherto, in the formula last written, n has had only three values, 0, 1, 2. We may ask whether further values are admissible. The wave-numbers of the three limits already concerned are

$$25227, \quad 24366{\cdot}5, \quad 23506,$$

with a common difference of 860·5. Continuing this sequence with the same common difference, we obtain the further limits

$$3833{\cdot}5, \quad 3711, \quad 3596, \quad 3489, \quad 3387, \quad 3290$$

towards the violet end of the spectrum. Using these as values of λ_0 in the formula

$$\lambda = \lambda_0 \frac{m^2}{m^2 - 1},$$

we find, corresponding to $m = 2$, the set of leading wave-lengths

$$5111, \quad 4948, \quad 4794, \quad 4652, \quad 4517, \quad 4388.$$

The first three are unknown in Campbell's spectrum, but the second three are all very characteristic Wolf-Rayet lines, λ 4652 being perhaps the most characteristic line in the spectrum.

It is curious that there should be apparently two groups of series, each group consisting of three, with a gap, which will be found later to be only apparent, between the same extent of wave-number as the limits of the series themselves. It is, however, difficult to confirm the existence of this second group until more observations have been taken. The Wolf-Rayet spectrum is not given by Campbell beyond λ 4000 into the violet, and even near λ 4000 – λ 4200 it is not very complete. No further test of any rigour can be applied, in fact, until we consider Wolf's spectrum, for the companion series

$$\lambda = \lambda_0 \frac{m^2}{m^2 - 0{\cdot}3286}$$

give leading wave-lengths

$$3802, \quad 3691, \quad 3585,$$

which are all below the observed range of Campbell's spectrum, as are also the remaining lines of the original series. We are compelled, therefore, to restrict our conclusion for the present to a statement that there is no evidence against the existence of three strong sets of series in this direction, whose leading lines are

$$\lambda\lambda 4652, \quad 4517, \quad 4388.$$

The last is the line whose origin from helium has been questioned already, on account of its strength and the absence of other helium lines.

Development from the Nebular Spectrum.

Not only the constant $q^2/4\omega^2$ already indicated in the formula is related to the atom of nebulium, but also the constants A and B themselves.

The two principal wave-lengths given in the nebular spectrum by the neutral atom of nebulium are λλ 5007 and 4363·5. If ω is the angular velocity in the atom, and C is the velocity of light, then in absolute units

$$5{\cdot}007.10^{-5} = \frac{2\pi C}{\omega},$$

$$4{\cdot}3635.10^{-5} = \frac{2\pi C}{q},$$

where $q/\omega = 1{\cdot}1465$ as before. Expressed in wave-numbers, the previous formulæ become

$$n = A\left(1 - \frac{q^2}{4\omega^2 m^2}\right),$$

where A takes a series of values with a common difference of 860·5. Thus three consecutive values are

$$25227, \quad 24366{\cdot}5, \quad 23506.$$

Now the wave-number of the nebular line λ 4363·5 is found to be 22917, and

$$23506 - \tfrac{1}{2}(22917) = 23506 - 11458{\cdot}5 = 12047{\cdot}5 = 14(860{\cdot}5),$$

so that 2(4363·5) is a member of the sequence of limits. Thus the series take the form

$$\lambda = \frac{4\pi C}{q + n\alpha}\left(\frac{m^2}{m^2 - q^2/4\omega^2}\right),$$

where now only the constant α remains arbitrary, or

$$\lambda = 2 \cdot \frac{5007}{q/\omega + n\alpha/\omega}\left(\frac{m^2}{m^2 - q^2/4\omega^2}\right),$$

where $\alpha/\omega = 0{\cdot}086086$ on calculation.

This is equivalent to

$$\lambda = \frac{2(4363{\cdot}5)}{1 + 3n/40}\left(\frac{m^2}{m^2 - q^2/4\omega^2}, \quad \frac{m^2}{m^2 - 1}\right),$$

and the passage from one formula to another in which n is different, appears to point irresistibly to an evolution of some kind, whose stages are marked by the value of n. The series are so bound up with the constants of the atom of nebulium that it seems certain that the Wolf-Rayet stars at least contain a substance in different forms, or a set of substances, whose atomic structure is very similar to that of nebulium. For if any change, apart from a change restricted to the nucleus, took part in the atom, the old value of q/ω would not be preserved. It seems, therefore, that the capacity of an atom to give out spectrum lines

which can be arranged in series in the ordinary way, must depend in some way on the more intimate structure of the nucleus, or on some feature in the atom which can change *without altering the forces between the component parts*. This is, of course, in no contradiction to Bohr's theory of spectra, in which the nature of the process contemplated in the binding of an electron by the positive nucleus is not treated from a mechanical point of view, but left arbitrary.

If this evolution is actually taking place in the early stars, some indication of its beginnings should appear in the nebular spectrum, and in particular there should be certain spectrum lines forming the connecting link between the nebulæ and the Wolf-Rayet stars. The present paper does not propose to enter into detail on this point, but only indicates certain directions in which this conclusion is partly supported, leaving a more complete discussion to follow later, when the general scope of these series has been laid down.

The successive limits of the series, starting with $n=0$, are, for increasing values of n,

8727, 8118, 7587, 7122, 6711, 6344·5, 6016, 5720, 5451·5, 5207·5, 4984, 4779, 4590, 4416, 4254.

Now, in series of the present type, no line can have a wave-length less than the limiting wave-length of the series, and observation of lines in nebulæ or Wolf-Rayet stars has not been made for wave-lengths greater than 6600. Accordingly, the first limit which can give rise to any known line is

$$\lambda_0 = 6344{\cdot}5,$$

and we may examine the lines

$$\lambda = 6344{\cdot}5\left(\frac{m^2}{m^2-1}, \quad \frac{m^2}{m^2-0{\cdot}3286}\right).$$

The leading members of the first series are

$$\lambda = 8459, \quad 7137, \quad 6767, \quad 6609$$

and are all outside the range of observation. Those of the second series are

$$\lambda = 6912, \quad 6584.$$

The first is outside the range, but the second is an important nebular line, first detected by Wright.* Recently, Merrill † has discovered that it is a Wolf-Rayet line also, and as this theory of the spectrum was developed some time ago, before this observation was made, the discovery has been a striking confirmation of the essential truth of this view of the spectra. We may conclude provisionally that, when this line is shown, we have to deal with a celestial body which is just passing from a nebular to a Wolf-Rayet stage. Further confirmation of this view will appear later.

When we examine the series whose limits come next in the

* *Lick. Obs. Bull.*, 183. † *Lick. Obs. Bull.*, 230.

sequence, we may expect to find Wolf-Rayet lines, with perhaps, as a preliminary, other nebular lines also, which become brighter in the older nebulæ, old nebulæ being defined as those in which the hydrogen and helium lines have become relatively brighter than those of nebulium. The most conspicuous example is the Orion nebula, which will be discussed in detail afterwards. The limits $\lambda_0 = 6016$, 5720 do not appear to give well-known lines, but an important line of the next series, with limit 5451·5, is 5813, a very marked Wolf-Rayet line. But, as this line is not accompanied by others, we cannot draw a definite conclusion in this case until the spectrum has been observed more completely near λ 6000.

A Subordinate Series.

There is reason to suspect at this stage that half the wave-length of one of the limits can be a limit also. This is confirmed in a later section, where some stronger series are indicated. But, as an illustration for the present, if we take the series involving λ 6583, which is a fairly strong line and therefore perhaps corresponds to a very stable atomic configuration, the half wave-lengths of the leading lines are 4229·5, 3568, 3456, where the last should be the strongest. It is significant that the Orion nebula contains a line at λ 423 which is not found in other nebulæ, and that a strong. line about λ 345 has been found by Palmer* in certain nebulæ, and is well known in novæ just before the final nebular stage.

Moreover, of the initial series of limits 8118, 7587, 7122, 6711, half the leading wave-lengths give $\lambda = 5412$, 4267, 4748, 4473. Three of these are known nebular lines, and perhaps the curious behaviour of λ 5412 is explicable in this way. This line is found in "earlier" nebulæ, disappears in Orion, and reappears in the Wolf-Rayet stars, where it is certainly a Pickering line. But in the earlier nebulæ it is much more probably due to the first evolution product of nebulium, which has already disappeared in Orion, where the line λ 4265 (Campbell), perhaps 4267 above, has become important. All these lines appear in the Wolf-Rayet stars. This suggested subordinate series is of importance later.

Further Discussion of Formulæ. A New Formula.

To summarise the results of the previous sections, it is suggested that evolution from nebulæ to Wolf-Rayet stars is accompanied by a progressive change in the spectra, due to an increase of n from one integral value to another in the formulæ

$$\lambda = \frac{(1 \text{ or } 2)5007}{(q_1/\omega) + 0{\cdot}08609n}\left(\frac{m^2}{m^2 - q^2/4\omega^2}\right),$$

where q/ω can have any value appropriate to the simple nebulium atom, and q_1/ω is one of its special values. This suggests, at once, the possible existence of series with other limits, obtained by

* *Lick. Obs. Bull.*, 183.

taking other special values for q_1/ω and leaving the remainder of the series unchanged.

The most important special value is, of course, $q_1/\omega = 1$, giving a series

$$\lambda = \frac{(1 \text{ or } 2)5007}{1 + 0{\cdot}08609n}\left(\frac{m^2}{m^2 - q^2/4\omega^2}\right).$$

Taking the series with 2 in the numerator, the successive limits are initially

10014, 9220, 8542, 7957, 7447,

where the first, corresponding to $n = 0$, may not be a limit of series. These are all, to that last written, incapable of giving visible lines, so that attention may be confined to the limits which are one-half of these, or

4610, 4271, 3978·5, 3723·5.

The first may be a weak nebular system. For the series

$$\lambda = 4610\frac{m^2}{m^2 - 1}$$

gives $\lambda = 5186$ for $m = 3$. The line 6146 for $m = 2$ is not known, but the spectrum is not well explored in this region. The second is a very strong Wolf-Rayet system. For the series

$$\lambda = 4271\frac{m^2}{m^2 - 1}$$

yields

$m = 2$,	$m = 3$,	$m = 4$.
$\lambda = 5694$,	4805,	4556.

The first is a characteristic Wolf-Rayet line, and the third is of usual occurrence. The second is not recorded by Campbell, but the theory can be tested by examining the other series

$$\lambda = 4271\frac{m^2}{m^2 - 0{\cdot}3286},$$

associated with this. The first line should be strong, for λ 5693 is intense, and the chemical system from which it originates must apparently possess some stability. The lines are

$m = 2$,	$m = 3$,
$\lambda = 4653$,	4433,

and the first is one of the most characteristic Wolf-Rayet lines. It is known, especially from reference to Wolf's recent paper, to be composite, so that the fact of its previous use in this paper is not an objection. The second line is not in Campbell's spectrum, but, like many others of previous series, exists in that of Wolf. The strength of these series indicates that the third series

$$\lambda = 4271\frac{m^2}{m^2 - 0{\cdot}2124}$$

should give a visible line, and the case $m = 2$ gives $\lambda = 4510$, where a good line is situated.

The system of lines with this limit is therefore well represented.

Summary.

The paper has shown that it is possible to represent, in a manner relating it essentially to the nebular spectrum, the typical spectrum of a Wolf-Rayet star, defining this typical or average spectrum by the results of Campbell's measurements. These measurements are not exhaustive, and it will be necessary later to discuss the formulæ with reference to the more detailed measurements published by Wolf during the present year for four separate types of Wolf-Rayet star. But the investigation presents features which tend very strongly to show the existence of a process of evolution from nebulæ in these stars, and the resemblance of the series which are put forward, to ordinary spectral series, indicates that some knowledge of the nature of such series may be derived, as the constants involved are no longer empirical.

Excluding the lines due to hydrogen and helium, practically every line in the spectrum comes into the scope of these series, and certain outstanding nebular lines also receive an interpretation. As an atom with four electrons in its ring, and some progressive modification in its more internal structure, is implied throughout, the results are not complete, and it is necessary to treat further the corresponding behaviour of charged atoms. As to the chemical nature of the progressive modifications of the original nebulium atom, no opinion is expressed. It is not unlikely that some at least are terrestrial elements, or proto-elements, for there are definite points in the sequence where the corresponding spectra are strong, and this appears to indicate atomic configurations of great stability.

The Light Curve of the Andromeda Nebula (N.G.C. 224).

By J. H. Reynolds.

§ 1. *Photographs.*—The negatives from which the following measures were made were taken with the 28-inch reflector (10 feet focal length) on 1912 October 7, 8, and 17, the exposures being 9 minutes, 16 minutes, 25 minutes, and 100 minutes respectively. Scale 1 millimetre = 69″.

§ 2. *Photomicrometer.*—The instrument used was one of the Hartmann model, supplied by Toepfer of Potsdam. In this type the portion of the negative to be measured is reflected into an eyepiece from below by a minute elliptical or slit-shaped mirror on a plane glass surface placed at an angle of 45°, the background against which it is seen being a photographic graduated scale of densities. Another eyepiece above this is used for locating the portion of the negative to be measured, the small mirror already mentioned serving as a point of reference. The stage carrying the negative is provided with a micrometer screw motion in one direction and a millimetre scale.

§ 3. *Calibration.*—The method of calibration adopted was based on the law of inverse squares. A board 12 feet long was grooved at the requisite distances to give a range of intensities of 1, 3, 5, 7,

10, 15, 20, 30, 40, 50, 60, 70, 80, 90, and 100, the last groove being placed at a distance of one foot from a small electric lamp, in front of which were placed ground glass and a screen with a small circular aperture. The light was reduced by means of photographic plates of various densities placed in front of the screen.

A plate of the same kind as those on which the nebular photographs were taken was cut into several small strips; these strips were placed in the grooves and exposed for 15 minutes, the light being reduced so that the strips when developed showed the same range of densities as the plates to be measured. The strips were then developed all together and their densities compared with the arbitrary photographic scale of the photomicrometer. From these data a curve was constructed, the abscissæ being the scale readings and the ordinates the light intensities on a scale of 100 light units. It was found that while the curve gave consistent readings up to 30 light units, beyond this point there were large discrepancies in the ratios given by different plates: this was, no doubt, due to the negatives being developed with pyro-soda and the calibration plates with metol. As, however, the curve gave satisfactory results up to 30 light units, the measures of the negatives were only carried up to this point.

§ 4. *The Measurement of the Negatives.*—The negative to be measured was placed face upwards on the micrometer stage and rotated until the major axis of the nebular ellipse was parallel with the screw motion: in this position the small nebula N.G.C. 221 was towards the observer. The slit-shaped mirror was used in all these measures, and the negative was adjusted so that the centre of the nucleus lay under this mirror in the finding eyepiece. The reading on the micrometer stage was taken and the photographic scale moved along until the density of the background was the same as the portion of the negative reflected by the slit. The negative was then moved by half a millimetre from its previous position and the process repeated: this was continued as long as the density readings differed perceptibly from the general density of the plate. This general density, arising from chemical and sky fogging of the film, varies with the negative, and the plate constant of density and its corresponding light intensity have to be recorded.

The series of readings on the arbitrary photographic scale were next converted into light units by means of the calibration curve, and the plate constant of intensity deducted, the result giving the light ratios of the portion of the nebula measured.

§ 5. The following table gives the resulting measures of the four negatives. The first column gives the distance from the centre of the nebula, the second the density reading on the arbitrary scale, the third the equivalent light units, the fourth the plate constant of intensity, and the fifth the corrected value. Details of the four negatives are given in this form. In the column C.R. are the combined readings from the four negatives, obtained by comparing the ratios; the column R.S. gives these readings reduced to a scale of 100, and the last column gives the

Distance from Centre in mm.	100 min. ex.				25 min. ex.				16 min. ex.				9 min. ex.				C.R.	R.S.	I.S. $(x+1)^2y$ = Constant.
	S.R.	L.U.	P.C.	C.V.	S.R.	L.U.	P.C.	C.V.	S.R.	L.U.	P.C.	C.V.	S.R.	L.U.	P.C.	C.V.			
6	25·3	12	9	3	..	...	...	...	...	...	...	...	...	...	...	...	3	2·5	2·3
5·5	25·3	12	9	3	...	..	...	..	...	...	...	...	...	...	...	...	3	2·5	2·3
5	26·0	13	9	4	...	...	...	...	...	...	...	...	...	...	...	...	4	3·4	3·3
4·5	26·2	14	9	5	...	...	...	...	...	...	..	...	...	...	...	...	5	4·2	4
4	26·2	14	9	5	...	...	...	...	...	...	...	...	...	...	...	...	5	4·2	4·9
3·5	26·8	16	9	7	...	...	...	...	...	...	...	...	...	...	...	...	7	5·9	6·2
3	27·1	18	9	9	22·6	8	6	2	21·8	7	6	1	...	..	...	...	9	7·5	8·1
2·5	28·4	24	9	15	23·4	9	6	3	22·2	7	6	1	...	...	...	...	15	12·5	11·1
2	29·0	28	9	19	24·4	10	6	4	23·2	8	6	2	...	...	...	...	19	15·9	16
1·5	...	...	...	...	25·8	13	6	7	24·3	10	6	4	...	...	...	...	37	30·9	25
1	...	...	...	...	27·2	19	6	13	25·0	11	6	5	22·0	7	5	2	58	48·4	44·4
·5	...	...	...	...	29·7	30	6	24	27 0	18	6	12	23·7	9	5	4	120	100	100
Centre	...	...	...	...	...	...	...	...	...	...	...	...	27·3	20	5	15	450	375	...
·5	...	...	...	...	29·7	30	6	24	27·4	26	6	14	23·7	9	5	4	130	108·4	100
1	..	..	...	...	27·3	19	6	13	25·6	12	6	6	22·0	7	5	2	62	51·7	44·4
1·5	...	...	...	...	25·3	12	6	6	24·6	10	6	4	...	...	...	...	35	29·2	25
2	29·4	29	9	20	24·3	10	6	4	23·9	9	6	3	...	...	...	...	20	16·7	16
2·5	28·3	24	9	15	23·7	9	6	3	22·5	7·5	6	1·5	...	...	...	...	15	12·5	11·1
3	27·4	20	9	11	23·0	8	6	2	22·5	7·5	6	1·5	...	...	...	...	11	9·2	8·1
3·5	26·9	16	9	7	...	...	...	...	...	...	...	...	...	...	...	...	7	5·9	6·2
4	26·1	14	9	5	...	...	...	..	...	...	.	...	...	..	...	...	5	4 2	4·9
4·5	25·9	13	9	4	...	...	...	...	...	...	...	...	...	...	...	...	4	3·4	4
5	25·8	12·5	9	3·5	...	...	...	...	...	...	..	...	...	...	...	...	3·5	2·9	3·3
5·5	25·2	12	9	3	...	...	...	...	...	...	...	...	...	...	...	...	3	2·5	2·8
6	25·2	12	9	3	...	...	...	...	...	..	...	...	...	...	..	...	3	2·5	2·3

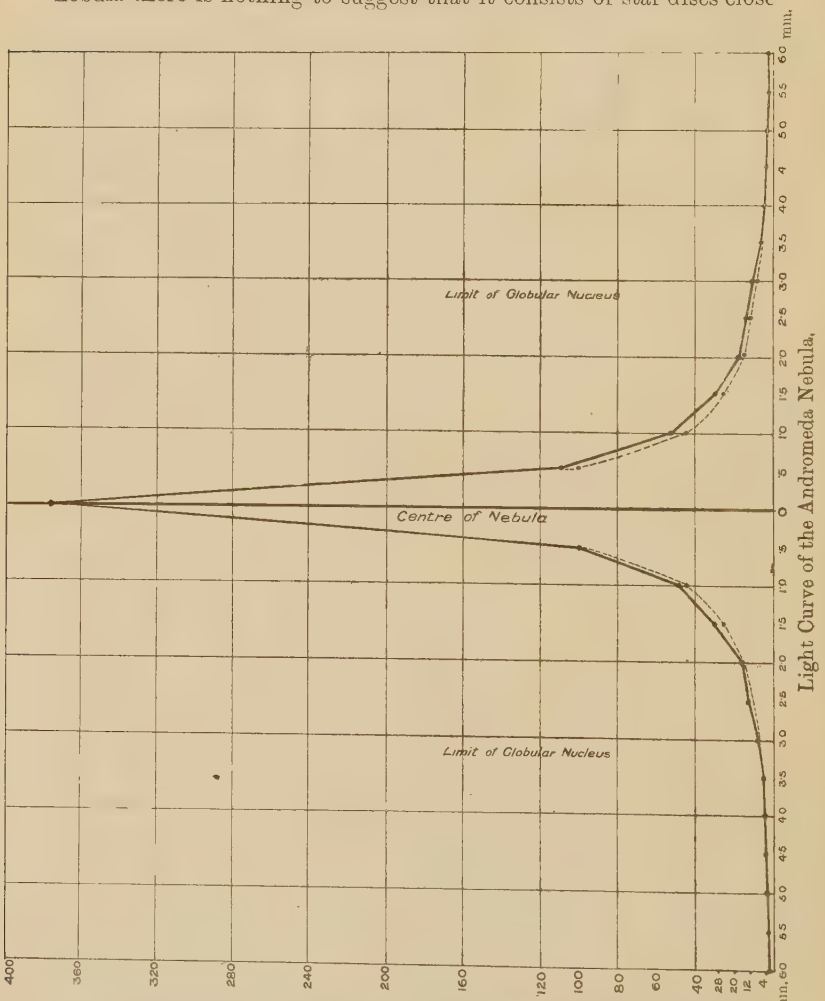
corresponding figure in a curve represented by the formula $(x+1)^2y = \text{Constant}$. Where the light ratios differ in any two negatives, the mean has been taken. Most of the readings are consistent within the limits of accuracy in gauging the density of the film. The only reading about which there is much uncertainty is that for the centre of the nebula, as there is no confirmatory reading on another negative, and as it has to be multiplied by a large factor to bring it into proportion with the figures given by the 100 minutes' exposure.

§ 6. The full line on the annexed diagram shows the light curve plotted from these values, and for comparison the curve $(x+1)^2y = \text{Constant}$ is also given with a dotted line. The remarkable similarity of the two curves on each side of the centre line must have some physical meaning, and it certainly seems to give some support to the hypothesis suggested in the *M.N.* of May 1912, that the light of the nebula is derived in some measure from a central star which is too much involved in nebulous matter to be visible as such. If the nebula were to consist of evenly distributed particles in a plane of uniform thickness, and if there was no absorption of the light by the particles, the light curve on this hypothesis would be the curve of inverse squares represented by the formula $x^2y = \text{Constant}$. But the actual composition of the nebula must be considered, and photographs show that it consists of a globular nucleus of about 7′ diameter, from which branch the two arms of the spiral. This globular formation would tend to flatten the hypothetical curve owing to the law of the cosines, and the depth of luminous material. The flattening would be most marked near the centre: in the outer regions the two curves $x^2y = \text{Constant}$ and $(x+1)^2y = \text{Constant}$ rapidly converge, both passing ultimately into almost parallel lines. The spiral arms stretch to a great distance from the nucleus; but as the intensity diminishes at a very slow rate beyond 6 mm. from the centre, and as it is difficult to detect with certainty smaller differences of density than one light unit, there is nothing to be gained by continuing the readings beyond this point. This almost imperceptible decrease in density in the regions of the nebula lying beyond 6 mm. from the nucleus is of course quite consistent with either of the curves.

§ 7. The usual alternative hypothesis as to the nature of the Andromeda Nebula is that it is a very distant galaxy of stars. This hypothesis was adopted in the days before photography, as it was found impossible to distinguish between genuine star clusters and nebulæ by visual observation. The discovery that the nebula gave a solar type of spectrum also seemed to point in the same direction, although, as far as this latter point is concerned, there is now good evidence that the spectrum is composite, and that bright lines, some of which correspond with the Wolf-Rayet lines, are superimposed on a solar absorption background.

It is often tacitly assumed that the defect of the eye as an optical instrument, which prevents it from seeing small stars

separately when they are massed together, is also shared by the photographic plate. There does not seem to be any evidence at all for such an assumption—in fact what evidence we have from the photography of star clusters is against it. In the negatives of the nebula there is nothing to suggest that it consists of star discs close



Light Curve of the Andromeda Nebula.

together, and even if we assume that the star discs are actually in contact all over the field, we must also make the very improbable assumption that the surface brightness of the stars follows the light curve.

My thanks are due to Mr. Stratton and Dr. Capstick for the assistance they have given me in this work.

Results of Micrometer Measures of Double Stars made with the 28-inch Refractor at the Royal Observatory, Greenwich, in the year 1912.

(*Communicated by the Astronomer Royal.*)

The measures were made with a bifilar position-micrometer on the 28-inch refractor, focal length 28 feet. The power generally employed was 670. When bright stars were observed a blue glass shade was usually employed to diminish the light and irradiation. The initals in the last column are those of the observers, viz. :—

S.C.	Mr. Chapman.	W.B.	Mr. Bowyer.
B.	Mr. Bryant.	H.F.	Mr. Furner.

Micrometric Observations of Double Stars at the Royal Observatory, Greenwich.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Distance.	No. of Nights.	Mags.		Epoch 1912.	Obs.
	h m	° ′	°	″					
β 862 ...	0 0	52 30	128·6	0·42	1	8·5	8·8	·969	W.B.
Σ 3056 AB	0 0	56 14	325·0	0·52	1	7·4	7·4	·969	W.B.
AC	...		0·1	23·31	1	7·4	9·0	·969	W.B.
Σ 3058 ...	0 1	60 10	51·4	12·39	2	7·7	9·2	·966	W.B.
Σ 3060 ...	0 1	72 25	119·0	3·38	2	8·5	8·7	·795	W.B.
			119·6	3·49	1	..		·857	H.F.
Σ 3061 ...	0 1	72 40	147·8	7·49	2	8·0	8·0	·966	W.B.
β 255 ...	0 7	62 5	92·1	0·58	2	7·5	8·5	·966	W.B.
OΣ 2 AB ..	0 9	63 31	33·2	0·48	3	6·9	8·3	·907	W.B.
AC ...	...		224·8	18·12	2	...		·966	W.B.
β 998 ...	0 9	83 55	111·9	1·05	1	8·5	8·6	·964	W.B.
β 1027 ...	0 10	68 57	183·6	1·50	2	7 7	10·5	·967	W.B.
Σ 19 ...	0 12	53 52	132·5	2·11	1	7·0	9·5	·969	W.B.
OΣ 4 AB ...	0 12	54 1	107·2	0·39	1	7·4	8·1	·969	W.B.
β 779 ...	0 23	66 55	252·7	0·92	1	8·5	9·0	·857	H.F.
Σ 42 ...	0 31	60 29	26·3	5·64	1	7·9	8·7	·800	W.B.
Σ 60 ...	0 44	32 40	247·2	6·54	1	4·0	7·6	·969	W.B.
β 495 ...	0 44	71 48	216·1	0·64	2	7·5	7·5	·803	W.B.
Σ 67 ...	0 47	79 53	0·8	1·83	2	8·3	9·0	·808	W.B.
Σ 73 ...	0 50	66 52	36·3	0·86	2	6·2	6·8	·983	W.B.
			33·1	0·86	1	...		·857	H.F.
β 867 ...	0 55	78 33	179·9	0·80	1	8·1	8·6	·857	H.F.
Σ 82 ...	0 56	81 0	303·0	1·67	2	8·3	9·3	·808	W.B.
			303·0	1·71	1	...		·857	H.F.
Σ 122 ...	1 22	86 56	329·9	6·43	1	7·9	9·0	·800	W.B.
Σ 178 ...	1 47	79 38	200·2	2·92	1	7·8	7·8	·800	W.B.

Micrometric Observations of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Distance.	No. of Nights.	Mags.		Epoch 1912.	Obs.
	h m	° ′	°	″					
Σ 180 ...	1 49	71 9	0·3	8·33	1	4·2	4·4	·857	H.F.
Σ 183 AB	1 50	61 38	354·6	0·45	1	7·5	8·2	·816	W.B.
AC	...		164·1	5·75	1	7·5	8·8	·816	W.B.
Σ 186 ...	1 51	88 36	218·3	0·90	1	7·2	7·2	·816	W.B.
Σ 194 ...	1 54	65 36	271·8	1·06	1	8·0	8·3	·857	H.F.
Holden 63	2 30	78 34	292·1	1·00	1	9·0	9·4	·997	W.B.
β 1315 ...	2 34	75 53	128·6	1·86	1	8·3	9·3	·997	W.B.
Hu. 1043 ...	2 36	74 57	240·3	2·73	1	9·0	9·5	·997	W.B.
Σ 305 ...	2 42	71 0	313·0	2·79	1	7·3	8·2	·997	W.B.
Σ 333 ...	2 54	69 1	201·9	1·28	1	5·7	6·0	·997	W.B.
Σ 381 ...	3 18	69 20	97·7	1·06	2	7·0	8·7	·981	W.B.
β 533 ...	3 30	58 37	42·0	0·72	1	7·0	7·0	·964	W.B.
OΣ 65 ...	3 45	64 41	202·9	0·45	2	6·5	6·8	·981	W.B.
Σ 499 ...	4 5	66 7	287·7	1·38	2	9·2	9·3	·493	W.B.
Ho. 328 ...	4 12	70 33	162·1	0·37	2	7·0	7·0	·981	W.B.
Σ 520 ...	4 14	79 50	117·2	0·81	2	8·0	8·0	·981	W.B.
OΣ 80 ...	4 17	47 47	184·1	0·42	2	6·5	7·0	·263	B.
OΣ 82 ...	4 17	75 10	111·3	0·52	1	7·0	9·0	·997	W.B.
Σ 535 ...	4 18	78 50	321·7	1·37	2	6·7	8·2	·017	W.B.
			321·5	1·47	2	...		·110	H.F.
Σ 546 ...	4 22	71 5	183·4	6·90	1	7·7	9·5	·090	H.F.
Hu. 1080 ...	4 24	74 3	253·4	0·37	2	6·5	7·5	·510	W.B.
Σ 562 ...	4 29	67 29	279·5	1·94	1	7·5	11·0	·129	H.F.
Σ 567 ...	4 31	70 41	323·4	1·65	2	8·5	9·0	·017	W.B.
			324·1	1·90	2	...		·110	H.F.
Hu. 305 ...	4 32	69 8	277·7	1·67	1	9·0	10·2	·022	W.B.
Ho. 333 ...	4 38	69 55	158·6	2·25	2	9·2	9·3	·017	W.B.
β 883 ...	4 46	79 5	194·2	0·37	1	7·0	7·0	·997	W.B.
Σ 620 ...	4 53	76 11	233·5	3·84	2	8·4	9·4	·110	H.F.
De. 6 ...	4 56	75 37	89·9	0·79	(2)	8·8	9·2	·017	W.B.
OΣ 95 ...	5 0	70 19	319·4	0·70	3	6·6	7·2	·035	W.B.
			322·5	0·98	1	...		·129	H.F.
OΣ 98 ...	5 3	81 37	153·9	0·75	1	6·0	6·8	·011	W.B.
			156·4	0·91	1	...		·129	H.F.
Σ 643 ...	5 3	81 42	114·4	2·50	1	8·5	8·5	·129	H.F.
Σ 648 ...	5 5	58 5	70·5	4·71	2	7·4	8·1	·083	W.B.
Ho. 334 ...	5 12	67 16	182·7	2·06	1	8·1	10·2	·071	W.B.
Ho. 226 ...	5 21	62 28	240·3	0·67	1	7·0	7·0	·071	W.B.

Micrometric Observations of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Distance.	No. of Nights.	Mags.		Epoch 1912.	Obs.
	h m	° ′	°	″					
Σ 716 ...	5 24	64 55	200·4	4·74	2	5·8	6·6	·083	W.B.
β 1267 ...	5 29	59 8	214·3	0·81	1	8·5	8·5	·071	W.B.
Σ 742 ...	5 31	68 4	262·2	3·35	2	7·2	7·8	·083	W.B.
			264·4	3·35	1	...		·129	H.F.
β 1007 ...	5 34	16 28	260·9	0·19	1	6·0	6·2	·255	B.
Σ 787 ...	5 41	68 43	69·6	1·03	3	8·1	8·5	·424	W.B.
OΣ 118 ...	5 43	69 9	322·3	0·50	1	6·2	7·7	·186	W.B.
Σ 795 ...	5 43	83 34	208·5	1·41	2	6·2	6·2	·504	W.B.
OΣ 123 ...	5 49	79 46	181·7	1·97	1	7·0	8·7	·068	H.F.
			178·0	1·92	3	...		·420	W.B.
Σ 816 ...	5 50	84 9	286·0	4·31	1	6·2	8·7	·068	H.F.
Σ 826 ...	5 54	91 20	122·1	1·75	1	8·2	9·2	·011	W.B.
Barnard 4	5 57	67 43	188·4	1·67	3	9·0	9·3	·089	W.B.
Σ 851 ...	6 3	86 42	29·1	2·71	1	8·2	8·7	·068	H.F.
β 1058 ...	6 5	60 59	263·5	0·30	2	7·2	7·5	·129	W.B.
			271·8	0·33	2	...		·285	B.
AG 107 ...	6 7	65 33	172·8	1·06	2	9·0	9·2	·017	W.B.
Σ 877 ...	6 8	75 21	262·7	5·45	2	7·2	7·7	·132	W.B.
Σ 888 ...	6 14	61 32	256·3	2·63	2	7·5	9·2	·083	W.B.
Hu. 702 ...	6 19	55 34	323·2	0·89	1	8·5	8·9	·249	S.C.
OΣ 140 ...	6 21	74 26	117·2	2·57	3	7·0	9·5	·057	W.B.
			115·7	2·70	1	...		·068	H.F.
AG 112 ...	6 22	65 25	207·2	2·83	3	9·0	9·1	·035	W.B.
			210·7	3·39	1	...		·249	S.C.
Σ 905 ...	6 23	49 49	127·7	2·48	1	8·0	10·0	·249	S.C.
Σ 932 ...	6 29	75 10	322·9	1·98	2	8·2	8·3	·017	W.B.
β 194 ...	6 30	51 56	287·3	1·16	1	8·0	8·5	·249	S.C.
AG 116 ...	6 31	51 42	30·2	2·70	1	8·8	9·1	·249	S.C.
Aitken 1053	6 35	64 50	340·8	1·02	1	8·8	10·2	·249	S.C.
Σ 930 AB	6 36	80 1	216·6	2·95	1	6·0	8·8	·997	W.B.
Σ 3118 ...	6 37	80 6	350·0	2·48	2	9·0	9·5	·510	W.B.
Σ 950² AB	6 37	80 1	47·5	3·52	1	9·0	10·5	·997	W.B.
OΣ 156 ...	6 42	71 43	291·5	0·44	3	6·5	7·0	·035	W.B.
			289·9	0·47	1	...		·315	B.
Σ 977 ...	6 49	41 19	128·4	2·28	1	8·0	9·5	·249	S.C.
Σ 981 ...	6 50	59 43	143·0	2·71	2	8·0	8·0	·079	W.B.
Σ 982 ...	6 50	76 42	157·7	6·13	1	5·4	7·7	·077	W.B.
β 1022 ...	6 55	62 36	313·5	0·36	2	8·5	8·5	·285	B.

Micrometric Observations of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Distance.	No. of Nights.	Mags.		Epoch 1912.	Obs.
	h m	° ′	°	″					
Ho. 342 ...	6 58	76 46	82·8	0·98	2	8·0	8·8	·129	W.B.
Σ 1009 ...	6 59	37 6	153·9	3·77	1	6·7	6·8	·249	S.C.
Hu. 454 ...	7 0	68 11	212·5	1·94	1	9·1	11·3	·071	W.B.
Hu. 836 ...	7 1	56 48	305·9	1·33	1	9·1	11·3	·249	S.C.
Hu. 618 ...	7 5	38 28	121·8	1·59	1	8·8	10·8	·249	S.C.
Σ 1037 ...	7 7	62 37	288·8	0·49	3	7·1	7·1	·232	W.B.
OΣ 170 ...	7 13	80 32	105·9	1·58	2	7·5	7·5	·129	W.B.
Σ 1068 ...	7 15	76 27	348·1	3·72	2	8·3	9·0	·192	W.B.
Σ 1081 ...	7 19	68 22	228·2	1·42	2	7·8	8·5	·184	W.B.
Hu. 706 ...	7 20	69 53	230·3	0·59	1	9·5	9·5	·203	W.B.
AG 140 ...	7 21	67 44	171·6	1·96	2	8·7	10·3	·244	W.B.
Σ 1092 ...	...		72·8	3·07	1	8·0	9·8	·249	S.C.
Σ 1093 ...	7 23	39 49	146·1	0·90	1	8·2	8·2	·249	S.C.
Ma. 2 ...	7 29	46 46	120·8	3·13	1	9·0	9·0	·249	S.C.
OΣ 174 ...	7 30	46 46	83·9	2·12	1	6·5	8·1	·249	S.C.
Ku. 30 ...	7 30	55 29	112·1	3·29	1	9·2	9·6	·249	S.C.
Σ 1119 ...	7 33	56 5	349·7	3·10	1	8·0	9·3	·249	S.C.
Ho. 245 ...	7 34	91 15	185·1	0·45	1	8·0	8·0	·315	B.
Σ 1126 ...	7 35	84 34	147·9	1·16	1	7·2	7·5	·181	W.B.
			152·8	1·18	1	...		·244	H.F.
Bird 2 ...	7 35	84 31	186·9	0·70	1	9·1	9·2	·244	H.F.
AG 142 ...	7 36	66 36	17·5	1·63	3	8·8	10·0	·259	W.B.
Σ 1130 ...	7 37	80 5	173·0	1·10	1	8·4	8·9	·181	W.B.
			176·7	1·13	1	...		·244	H.F.
Ho. 247 ...	7 41	68 39	124·0	0·39	3	7·5	8·0	·266	W.B.
			129·6	0·40	2	...		·285	B.
AG 145 ...	7 48	80 39	249·9	4·59	1	9·2	9·6	·244	H.F.
OΣ 182 ...	7 48	86 22	32·4	0·92	1	7·0	7·5	·181	W.B.
			29·2	1·03	1	...		·244	H.F.
AG 146 ...	7 52	39 30	283·8	3·41	1	9·1	9·1	·249	S.C.
Σ 1161 ...	7 53	43 8	191·9	3·03	1	7·8	9·7	·249	S.C.
OΣ 185 ...	7 53	88 38	30·4	0·37	2	6·8	7·0	·285	B.
Σ 1171 ...	7 56	66 10	328·7	2·87	1	6·2	10·7	·249	S.C.
Σ 1175 ...	7 58	85 36	231·0	1·53	1	7·8	9·7	·181	W.B.
			230·4	1·60	1	...		·244	H.F.
OΣ 187 ...	7 58	56 43	241·7	0·26	1	6·9	7·5	·315	B.
Hu. 713 ...	7 58	40 27	130·7	4·76	1	9·0	9·0	·249	S.C.
β 23 ...	7 58	86 39	184·4	2·50	1	8·2	12·0	·244	H.F.

Micrometric Observations of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Distance.	No. of Nights.	Mags.		Epoch 1912.	Obs.
	h m	° ′	°	″					
β 581 ...	7 58	77 27	278·9	0·17	1	8·0	8·0	·315	B.
Hu. 850 ...	8 3	52 10	356·4	0·74	3	8·7	9·1	·306	W.B.
Σ 1196 AB	8 7	72 4	319·2	1·03	2	5·0	5·7	·236	W.B.
AC	...		108·8	5·07	2	...	5·5	·236	W.B.
BC	...		117·7	5·95	2	5·7	5·5	·236	W.B.
Σ 1197 ...	8 7	60 1	102·2	1·81	2	8·6	9·0	·298	W.B.
Σ 1202 ...	8 9	78 52	317·3	2·30	2	7·7	9·8	·192	W.B.
β 204 ...	8 9	79 20	298·4	0·87	1	7·1	10·1	·244	H.F.
β 1244 ...	8 9	87 44	37·3	0·84	1	7·9	8·1	·244	H.F.
Σ 1205 ...	8 12	33 16	178·5	1·33	1	8·5	8·8	·249	S.C.
Σ 1211 ...	8 12	54 43	124·3	0·78	2	8·7	9·2	·305	W.B.
β 1320 ...	8 15	72 43	4·6	5·01	1	9·5	9·8	·244	H.F.
Σ 1216 ...	8 17	91 19	211·0	0·38	1	7·5	8·2	·315	B.
Hu. 856 ...	8 20	52 19	266·8	0·28	2	7·5	8·2	·305	W.B.
Σ 1223 ...	8 21	62 46	216·9	4·86	2	6·0	6·5	·297	W.B.
AG 153 ...	8 30	55 5	88·2	3·50	1	9·0	9·1	·249	S.C.
AG 154 ...	8 32	66 26	182·3	1·51	2	9·1	9·3	·236	W.B.
Σ 1244 ...	8 32	47 53	0·0	3·76	1	8·2	9·8	·249	S.C.
Hu. 858 ...	8 33	77 43	158·3	0·81	1	9·1	9·8	·244	H.F.
Ku. 32 ...	8 36	70 24	167·0	1·77	2	8·4	10·2	·241	W.B.
			174·8	2·23	1	...		·244	H.F.
β 858 ...	8 36	69 12	108·4	0·68	1	9·1	9·8	·244	H.F.
Σ 1265 ...	8 39	76 3	310·3	5·59	1	8·4	10·8	·244	H.F.
Kr. 30 ...	8 40	31 58	116·9	2·19	1	9·5	9·5	·249	S.C.
Σ 1273 AB	8 42	83 14	227·0	0·24	1	3·8	7·8	·315	B.
			220·0	0·28	2	...		·325	W.B.
$\frac{AB}{2}$. C			240·2	3·46	2		12·5	·311	W.B.
Σ 1279 ...	8 44	50 5	88·2	1·47	2	8·3	8·3	·305	W.B.
AG 157 ...	8 45	66 32	76·5	1·74	2	9·3	9·5	·228	W.B.
Perrotin ...	8 46	81 20	349·1	0·80	1	7·5	8·7	·244	H.F.
Σ 1287 ...	8 47	77 32	92·9	1·85	2	8·0	10·3	·241	W.B.
Σ 1291 ...	8 49	59 5	320·8	1·47	1	5·9	6·4	·329	S.C.
Aitken ...	8 50	63 27	253·8	0·27	1	6·9	8·0	·329	W.B.
Hu. 718 ...	8 55	57 14	191·9	0·50	3	8·7	8·9	·304	W.B.
Σ 1300 ...	8 56	74 22	193·7	5·03	2	8·7	8·8	·271	W.B.
			196·4	5·18	1	...		·279	H.F.
Ait. 554 ...	9 0	60 55	198·2	0·70	2	8·5	10·5	·302	W.B.

Micrometric Observations of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Distance.	No. of Nights.	Mags.		Epoch 1912.	Obs.
	h m	° ′	°	″					
Σ 3121 ...	9 13	61 2	183·7	0·30	1	8·0	8·3	·329	W.B.
Σ 1338 ...	9 15	51 10	174·2	1·58	2	7·0	7·2	·296	W.B.
Σ 1339 ...	9 15	52 53	72·5	1·35	2	8·5	9·5	·304	W.B.
AG 165 ...	9 18	67 27	194·8	1·16	2	9·1	9·3	·228	W.B.
OΣ 201 ...	9 19	61 43	216·6	1·51	3	7·5	9·0	·288	W.B.
Σ 1348 ...	9 20	83 16	319·5	1·71	2	7·5	7·6	·277	W.B.
			318·0	1·80	1	...		·279	H.F.
Σ 1355 ...	9 23	83 22	335·6	2·30	2	7·2	7·2	·277	W.B.
			337·4	2·43	2	...		·299	H.F.
Σ 1356 ...	9 24	80 33	121·0	1·01	2	6·2	7·0	·256	W.B.
			120·8	0·95	1	...		·279	H.F.
Σ 1372 ...	9 32	73 22	56·7	0·58	2	8·2	8·3	·307	W.B.
			109·2	0·25	1	7·7	7·8	·301	W.B.
Ho. 369 ...	9 46	53 5	123·3	0·16	1	...		·315	B.
Ait. 344 ...	9 46	60 18	33·7	0·51	1	8·6	9·2	·312	W.B.
OΣ 208 ...	9 46	35 30	311·0	0·39	1	5·0	5·6	·315	B.
AC 5 ...	9 48	97 40	68·0	0·45	2	5·5	5·8	·285	B.
Σ 1397 ..	9 52	64 31	103·6	1·42	2	8·5	10·3	·293	W.B.
Σ 1413 ...	10 7	73 13	95·7	1·95	3	8·9	8·9	·210	W.B.
OΣ 215 ...	10 11	71 49	203·1	0·99	2	7·0	7·2	·288	W.B.
Hu. 875 ...	10 13	52 2	66·0	0·83	1	7·0	9·8	·405	W.B.
Σ 1423 ...	10 14	68 59	53·4	1·50	2	8·6	9·3	·228	W.B.
Σ 1424 ...	10 15	69 42	114·1	3·71	2	2·0	3·5	·274	W.B.
Σ 1426 ...	10 16	83 7	290·0	0·62	1	7·5	8·0	·181	W.B.
Σ 1429 ...	10 20	64 55	243·0	0·61	3	8·3	8·3	·252	W.B.
OΣ 217 ...	10 22	72 19	149·0	0·83	4	7·3	7·8	·266	W.B.
Hu. 879 ...	10 23	52 50	240·1	0·33	1	4·0	6·5	·315	B.
OΣ 220 ...	10 24	79 23	68·1	1·34	2	7·1	9·0	·244	W.B.
Σ 1439 ...	10 25	68 44	111·3	1·65	2	8·0	8·5	·178	W.B.
Σ 1450 ...	10 30	80 53	154·9	2·21	2	6·0	8·7	·244	W.B.
OΣ 224 ...	10 35	80 41	284·5	0·46	1	7·2	9·2	·312	W.B.
OΣ 227 ...	10 37	78 47	340·1	0·60	3	7·5	8·5	·265	W.B.
OΣ 228 ...	10 42	66 57	184·9	0·41	4	7·2	8·1	·329	W.B.
Ho. 376 ...	10 48	66 19	218·3	3·21	1	8·8	10·0	·394	W.B.
Hu. 567 ...	10 48	67 23	181·8	0·71	2	9·3	10·0	·353	W.B.
Hu. 568 ...	10 50	68 48	30·6	0·48	1	9·3	9·8	·394	W.B.
Σ 1487 ...	10 51	64 46	107·7	6·74	2	5·0	7·0	·400	W.B.
Σ 1501 ...	10 57	58 42	5·2	1·70	2	9·0	9·3	·348	W.B.

Micrometric Observations of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Distance.	No. of Nights.	Mags.		Epoch 1912.	Obs.
	h m	° ′	°	″					
Ho. 47 BC	10 58	53 51	328·3	0·75	1	9·0	9·0	·394	W.B.
Ho. 378 ...	11 0	51 6	226·3	0·60	1	8·0	8·2	·394	W.B.
Σ 1517 ...	11 9	69 23	87·3	0·32	3	7·3	7·3	·250	W.B.
OΣ 232 ...	11 10	51 56	235·1	0·60	3	7·0	7·8	·367	W.B.
Σ 1523 ...	11 13	57 57	118·6	2·95	2	4·0	4·9	·288	W.B.
Σ 1527 ...	11 14	75 14	18·7	3·23	1	6·9	8·1	·312	W.B.
Σ 1536 ...	11 19	78 58	49·7	2·00	1	3·9	7·1	·175	W.B.
Σ 1537 ...	11 20	68 53	355·6	2·35	2	7·6	9·6	·225	W.B.
OΣ 234 ...	11 26	48 13	164·0	0·39	1	7·0	7·4	·394	W.B.
OΣ 235 ...	11 27	28 21	251·4	0·42	3	6·0	7·3	·388	B.
Hu. 1134 ...	11 27	53 15	130·5	0·15	2	7·0	7·0	·351	B.
			106·6	0·19	1	...		·394	W.B.
Hu. 727 ...	11 28	40 3	13·2	1·09	1	8·8	9·2	·394	W.B.
Σ 1554 ...	11 31	76 39	82·4	0·68	3	8·8	8·8	·252	W.B.
Σ 1555 ...	11 32	61 43	174·7	0·26	4	6·4	6·8	·320	W.B.
			185·8	0·25	1		11·0	·430	B.
OΣ 237 ..	11 34	48 21	262·2	1·31	3	7·4	9·0	·400	W.B.
OΣ 241 ...	11 52	54 3	128·4	1·44	2	6·5	8·4	·400	W.B.
Ait. 681 ...	11 59	64 24	122·6	0·48	2	8·9	9·3	·291	W.B.
Ait. 682 ...	12 0	65 23	331·9	0·39	1	7·5	9·0	·307	W.B.
Σ 1601 ...	12 2	50 40	311·9	2·21	1	8·5	9·7	·405	W.B.
Σ 1606 ...	12 6	49 36	327·2	0·97	2	6·3	7·0	·400	W.B.
Σ 1613 ...	12 8	53 44	10·3	1·33	2	8·5	8·8	·400	W.B.
Σ 1619 ...	12 11	96 45	277·9	7·39	1	7·5	7·8	·312	W.B.
Σ 1634 ...	12 16	66 35	148·2	0·68	1	8·1	9·9	·413	H.F.
Σ 1639 ...	12 20	63 55	340·2	0·56	2	6·7	7·9	·310	W.B.
			345·3	0·74	2	...		·318	S.C.
			350·9	0·68	1	...		·430	B.
Σ 1643 ...	12 23	62 28	34·0	2·03	2	8·7	9·2	·358	W.B.
			37·4	2·00	1	...		·413	H.F.
β 1324 ...	12 25	59 59	222·5	2·53	1	9·3	9·9	·405	W.B.
			224·2	2·51	1	...		·413	H.F.
Σ 1647 ...	12 26	79 47	227·2	1·35	2	7·5	7·8	·318	S.C.
			223·4	1·44	2	...		·342	W.B.
			221·9	1·26	1	...		·435	H.F.
AG 179 ...	12 28	66 30	135·5	1·06	1	9·0	10·0	·413	H.F.
Σ 1661 ...	12 31	78 6	241·5	2·43	1	8·5	8·5	·435	H.F.

Micrometric Observations of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Distance.	No. of Nights.	Mags.	Epoch 1912.	Obs.
	h m	° ′	°	″				
Σ 1663 ...	12 33	68 18	97·6	0·68	2	7·8 8·7	·356	W.B.
			97·8	0·84	2	...	·424	H.F.
Σ 1668 ...	12 36	84 41	193·1	1·64	2	7·5 8·0	·342	W.B.
Σ 1670 AB	12 37	90 57	327·2	5·91	1	3·0 3·0	·326	S.C.
AC	12 37	90 57	241·9	84·60	1	3·0 14·5	·326	S.C.
AD	...		268·8	120·12	1	3·0 11·6	·326	S.C.
Σ 1674 ...	12 39	81 57	172·9	2·22	2	8·5 9·2	·342	W.B.
Hu. 893 ...	12 43	77 14	32·0	1·63	1	9·1 9·1	·307	W.B.
Σ 1680 ...	12 45	67 44	339·9	2·95	1	8·8 11·0	·413	H.F.
Hu. 894 ...	12 49	76 20	324·5	0·84	1	9·1 9·1	·435	H.F.
Σ 1687 ...	12 49	68 16	82·1	1·09	1	5·2 8·0	·435	H.F.
OΣ 256 ...	12 52	90 28	74·3	0·72	1	7·2 7·6	·430	B.
Σ 1699 ...	12 54	62 2	183·7	1·60	3	7·8 7·8	·313	W.B.
			182·7	1·70	1	...	·309	S.C.
			183·1	1·49	1	...	·435	H.F.
Σ 1700 ...	12 54	62 24	83·4	1·19	1	8·2 10·0	·435	H.F.
β 112 ...	12 56	71 9	294·5	1·97	1	9·6 10·0	·435	H.F.
Σ 1711 ...	12 58	76 3	343·2	1·00	1	8·5 9·0	·307	W.B.
			351·0	0·87	1	...	·435	H.F.
β 929 ...	12 59	93 11	212·5	0·49	2	6·0 6·3	·373	B.
			218·3	0·70	1	...	·326	S.C.
β 341 ...	12 59	110 6	308·8	0·91	1	6·2 6·7	·326	S.C.
Σ 1714 ...	12 59	65 53	126·0	3·17	1	8·8 9·2	·312	W.B.
Σ 1728 ...	13 6	72 0	12·8	0·27	2	6·0 6·0	·314	W.B.
			12·1	0·25	2	...	·447	B.
OΣ 261 ...	13 8	57 27	343·5	1·69	2	6·9 7·4	·403	W.B.
Σ 1730 ...	13 8	52 37	332·9	1·92	1	8·4 10·1	·394	W.B.
Σ 1734 ...	13 16	86 35	186·2	1·30	1	7·2 7·9	·309	S.C.
Ait. 565 ...	13 18	62 21	24·0	0·68	1	8·7 10·3	·435	H.F.
Ho. 260 ...	13 19	60 18	329·0	0·75	2	8·3 8·5	·403	W.B.
OΣ 266 ...	13 24	73 49	345·2	2·21	1	7·3 7·8	·309	S.C.
			341·8	1·71	2	...	·378	W.B.
β 113 ...	13 25	78 3	211·1	1·64	1	8·5 11·0	·435	H.F.
OΣ 269 ...	13 29	54 38	219·2	0·24	1	6·5 7·0	·394	W.B.
			216·8	0·36	2	...	·447	B.
β 932 ...	13 30	102 45	106·0	0·26	2	6·1 6·6	·373	B.
Ku. 45 ...	13 30	74 16	297·1	0·88	1	9·8 10·0	·435	H.F.

Micrometric Observations of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Distance.	No. of Nights.	Mags.		Epoch 1912.	Obs.
	h m	° ′	°	″					
β 612 ...	13 35	78 48	356·0	0·19	1	6·0	6·0	·320	W.B.
			363·0	0·18	3	...		·403	B.
Hu. 645 ..	13 35	68 5	21·7	0·84	1	9·5	9·5	·435	H.F.
Σ 1781 ...	13 42	84 26	106·8	0·81	2	7·8	8·2	·346	W.B.
Σ 1785 ...	13 45	62 34	331·7	1·69	1	7·2	7·5	·309	S.C.
β 937 ...	13 53	55 8	118·2	0·79	1	8·1	8·3	·394	W.B.
Ait. 687 ...	13 56	61 8	308·6	0·95	1	9·2	9·3	·435	H.F.
Ait. 569 ...	13 57	64 13	100·8	0·51	1	9·0	9·3	·320	W.B.
			114·0	0·69	1	...		·435	H.F.
Ait. 346 ...	14 4	64 51	336·1	0·81	1	8·6	9·6	·435	H.F.
OΣ 276 ...	14 4	52 50	193·6	0·51	3	7·5	8·3	·475	W.B.
OΣ 277 ...	14 8	60 52	2·1	0·44	3	7·8	8·0	·465	W.B.
β 224 ...	14 9	77 1	63·3	0·59	1	8·9	9·3	·435	H.F.
OΣ 278 ...	14 9	45 24	72·9	0·33	3	7·5	7·7	·459	B.
			67·5	0·39	2	...		·460	W.B.
Σ 1819 ...	14 11	86 27	349·7	1·30	1	7·9	8·0	·383	W.B.
Howe 32 ...	14 12	66 13	191·5	4·78	2	9·1	9·5	·460	W.B.
Σ 1834 ...	14 17	41 5	9·11	0·16	3	7·1	7·2	·459	B.
Σ 1835 ...	14 19	81 8	188·1	6·21	1	5·5	6·8	·320	W.B.
β 1111 ...	14 19	81 8	70·1	0·27	1	8·4	8·4	·320	W.B.
Ait. 1105 ...	14 23	58 58	190·7	0·95	1	9·1	9·4	·435	H.F.
			185·0	0·89	3	...		·486	W.B.
Ait. 570 ...	14 28	62 55	113·4	0·22	4	6·3	6·5	·423	B.
			116·8	0·24	2	...		·412	W.B.
Σ 3086 ...	14 28	72 18	271·0	6·01	2	9·0	10·0	·378	W.B.
Σ 1863 ...	14 35	38 2	87·6	0·65	3	7·1	7·4	·459	B.
Σ 1864 ...	14 35	73 8	103·9	5·84	2	7·9	6·0	·482	W.B.
Σ 1865 ...	14 37	75 53	137·1	0·51	2	3·5	3·9	·374	W.B.
			136·6	0·63	2	...		·373	B.
Σ 1866 ...	14 37	80 6	197·5	0·76	2	8·2	8·2	·482	W.B.
Σ 1877 ...	14 41	62 33	337·7	3·10	2	3·0	6·3	·337	S.C.
Σ 1879 ...	14 42	79 58	121·0	0·85	1	7·7	8·5	·326	S.C.
			119·1	0·79	2	...		·348	W.B.
			121·9	0·68	2	...		·447	B.
OΣ 285 ...	14 42	47 15	104·0	0·37	3	7·1	7·6	·459	B.
Σ 1883 ...	14 44	83 41	237·3	0·64	1	7·0	7·0	·326	S.C.
			232·1	0·33	2	...		·447	B.
Σ 1888 ...	14 47	17 32	127·7	2·21	1	4·7	6·6	·539	S.C.

Micrometric Observations of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle	Distance.	No. of Nights.	Mags.	Epoch 1912.	Obs.
	h m	° ′	°	″				
OΣ 287 ...	14 48	44 42	322·1	0·87	3	7·5 7·6	·522	W.B.
OΣ 288 ...	14 49	73 55	189·6	1·67	1	6·4 7·1	·309	S.C.
			187·8	1·66	2	...	·379	W.B.
Hu. 745 ...	15 0	69 48	22·7	0·51	1	7·5 9·0	·435	H.F.
Σ 1908 ...	15 1	55 11	144·8	1·42	2	8·2 9·2	·515	W.B.
Σ 1907 ...	15 1	78 1	183·7	1·12	2	8·5 8·7	·379	W.B.
			187·9	1·05	1	...	·435	H.F.
Σ 1910 ...	15 3	80 26	211·6	4·58	1	7·0 7·0	·435	H.F.
Ait. 690 ...	15 5	61 32	359·8	0·84	2	9·0 9·5	·451	W.B.
Ait. 1116 ...	15 7	79 32	25·0	0·48	2	8·1 8·1	·447	B.
Ho. 60 ...	15 10	54 57	27·2	0·49	2	7·5 7·6	·460	W.B.
Σ 1932 ...	15 15	62 51	354·7	0·56	3	5·6 6·1	·359	W.B.
			363·2	0·68	3	...	·459	B.
			356·9	0·71	1	..	·441	S.C.
Σ 1937 ..	15 20	59 24	41·0	0·95	2	5·5 6·0	·529	W.B.
			36·4	1·14	2	...	·490	S.C.
Σ 1941 ...	15 22	63 3	225·8	1·14	2	8·7 8·7	·318	S.C.
			223·3	1·41	2	...	·529	W.B.
Σ 1944 ...	15 23	83 35	320·8	0·99	1	7·5 8·1	·435	H.F.
Ait. 82 ...	15 23	65 46	312·1	0·88	3	8·5 9·3	·359	W.B.
			323·2	0·75	1	...	·435	H.F.
Σ 1950 ...	15 26	64 11	91·3	3·07	2	6·7 8·2	·377	W.B.
			96·9	3·62	1	...	·309	S.C.
			92·0	3·46	1	...	·435	H.F.
Hu. 1163 ..	15 27	51 32	268·4	0·62	1	8·2 8·5	·526	W.B.
Hu. 577 ...	15 29	69 57	36·5	0·33	2	8·0 8·0	·348	W.B.
			33·8	0·39	3	...	·459	B.
Σ 1957 ...	15 32	76 47	153·0	0·97	2	7·9 9·6	·379	W.B.
AG 197 ...	15 36	68 26	126·1	3·18	2	9·0 9·1	·457	W.B.
Σ 1965 ...	15 36	53 4	124·0	6·62	2	4·9 5·9	·424	S.C.
Hu. 580 ...	15 38	70 3	55·7	0·22	2	5·0 5·0	·348	W.B.
			68·2	0·22	3	...	·459	B.
Σ 1967 ...	15 39	63 25	110·2	0·63	1	4·0 7·0	·320	W.B.
			119·7	0·75	1	...	·326	S.C.
			115·1	0·72	1	...	·471	H.F.
			114·7	0·63	2	...	·474	B.
β 619 ...	15 39	76 3	3·5	0·54	1	6·9 7·7	·471	H.F.

Micrometric Observations of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Distance.	No. of Nights.	Mags.		Epoch 1912.	Obs.
	h m	° ′	°	″					
Σ 1967 ...	15 39	63 25	111·2	0·63	1	4·0	7·0	·531	W.B.
			114·7	0·84	2	...		·490	S.C.
β 621 ...	15 45	45 13	52·5	0·45	1	7·5	8·0	·526	W.B.
Σ 1988 ...	15 53	77 15	79·7	2·38	2	7·5	8·2	·378	W.B.
OΣ 303 ...	15 57	76 28	149·2	0·79	3	7·4	7·9	·340	W.B.
			148·9	1·08	2	...		·433	S.C.
			152·6	0·89	1	...		·471	H.F.
Σ 1998 ...	15 59	101 8	346·5	0·71	3	4·9	5·2	·459	B.
Σ 2000 ...	15 59	75 45	227·4	2·36	1	8·2	9·0	·471	H.F.
β 812 ...	16 3	72 52	124·1	0·77	2	8·2	8·3	·378	W.B.
			117·6	0·85	2	...		·433	S.C.
			117·9	0·90	1	...		·471	H.F.
β 949 ...	16 4	99 52	14·7	0·13	2	7·6	7·7	·474	B.
Σ 2021 ...	16 9	76 14	340·8	3·93	1	6·7	7·9	·471	H.F.
Σ 2022 ...	16 9	63 6	138·0	2·47	3	6·2	9·8	·472	W.B.
OΣ 309 ...	16 16	48 7	249·5	0·44	1	7·5	7·7	·485	B.
Hu. 481 ...	16 17	66 48	226·5	0·61	1	7·3	9·2	·471	H.F.
β 951 ...	16 20	56 26	39·9	0·84	2	8·2	8·7	·655	W.B.
Σ 2049 ...	16 24	63 49	208·6	1·27	3	6·5	7·5	·391	S.C.
			204·3	1·22	2	...		·374	W.B.
			201·8	1·20	1	...		·471	H.F.
β 813 ...	16 24	63 16	164·9	1·07	2	8·4	8·4	·374	W.B.
			170·7	1·00	1	...		·471	H.F.
			168·1	0·98	1	...		·539	S.C.
Σ 2052 ...	16 25	71 24	79·7	0·81	2	7·5	7·5	·374	W.B.
			83·0	1·00	1	...		·471	H.F.
			82·3	0·99	1	...		·539	S.C.
Σ 2057 ...	16 26		266·0	5·01	2	9·7	9·9	·379	W.B.
Σ 2055 ...	16 26	87 49	73·0	0·93	1	4·0	6·1	·528	H.F.
Σ 3105 ...	16 27	96 50	31·0	0·35	3	7·7	7·7	·459	B.
Hu. 1173 ...	16 27	55 55	71·4	0·37	1	8·4	8·7	·485	B.
Σ 2058 ...	16 28	70 30	346·2	1·85	2	9·0	9·5	·374	W.B.
OΣ 313 ...	16 30	49 42	144·5	1·08	1	7·2	7·8	·485	B.
Σ 2061 ...	16 30	58 54	24·7	2·68	2	7·1	9·1	·534	W.B.
Hu. 485 ...	16 35	67 4	276·6	4·29	1	8·8	13·0	·427	W.B.
OΣ 314 ...	16 35	69 22	234·1	3·63	1	7·2	10·1	·471	H.F.
			233·5	3·62	2	...		·491	W.B.

Micrometric Observations of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Distance.	No. of Nights	Mags.	Epoch 1912.	Obs.
	h m	° ′	°	″				
Hu. 486 ...	16 36	67 6	146·4	1·09	1	9·0 10·0	·471	H.F.
			142·8	1·09	2	...	·482	W.B.
Hu. 487 ...	16 38	67 59	23·2	0·51	3	9·0 9·0	·459	B.
Σ 2084 ...	16 38	58 15	125·0	1·14	2	3·0 6·5	·616	B.
			127·2	1·31	4	...	·538	W.B.
Σ 2091 ...	16 39	48 39	308·0	1·05	3	7·5 8·0	·688	W.B.
Σ 2094 ...	16 40	66 19	259·2	1·59	1	7·3 7·6	·539	S.C.
			256·6	1·44	2	...	·375	W.B.
			262·7	1·30	1	..	·528	H.F.
De. 15 ...	16 41	46 21	270·6	0·51	1	8·1 8·6	·767	W.B.
Σ 2097 ...	16 42	54 6	83·9	1·93	2	8·5 8·7	·488	W.B.
Σ 2106 ...	16 47	80 26	278·1	0·19	3	6·7 8·4	·459	B.
Σ 2107 ...	16 48	61 11	22 7	0·76	1	6·5 8·0	·441	S.C.
			19·5	0·47	3	...	·448	W.B.
			18·7	0·43	2	...	·474	B.
AC ...			309·5	82·37	1	...	·441	S.C.
β 821 ...	16 48	58 0	311·4	1·25	1	8·4 8·9	·539	S.C.
Ait. 350 ...	16 50	60 45	133·1	0·42	2	9·0 9·0	·627	B.
OΣ.318 ...	16 53	75 43	253·9	2·58	1	6·7 9·3	·471	H.F.
OΣ 319 ...	16 54	74 43	60·4	0·99	2	7·5 8·5	·379	W.B.
			65·4	0·99	1	...	·471	H.F.
OΣ 321 ...	16 55	75 34	8·3	0·77	1	7·7 8·7	·471	H.F.
Σ 2118 ...	16 56	24 50	84·2	0·40	2	6·4 6·9	·627	B.
OΣ 322 ...	16 57	52 57	202·4	1·31	3	7·0 9·8	·688	W.B.
Σ 2114 ...	16 58	81 26	165·3	1·17	2	6·2 7·4	·405	W.B.
Perry ...	17 1	70 17	231·8	1·85	2	6·9 10·2	·405	W.B.
β 357 ...	17 1	79 20	295·9	1·19	2	8·3 10·0	·405	W.B.
Σ 2120 ...	17 1	61 48	240·9	9·37	1	6·4 9·2	·490	H.F.
			240·8	9·65	1	...	·531	W.B.
Ait. 228 ...	17 3	63 22	186·9	0·64	3	9·0 9·2	·393	W.B.
Hu. 1176 ...	17 5	53 57	110·0	0·16	3	6·0 6·0	·689	B.
β 1118 ...	17 5	105 36	239·9	0·42	1	3·4 3·9	·463	B.
Σ 2135 ...	17 8	68 40	179·5	7·51	2	7·1 8·4	·436	W.B.
			176·8	7·42	1	...	·490	H.F.
Σ 2137 ...	17 10	73 56	144·0	4·05	2	8·2 9·2	·534	W.B.
Σ 2140 ...	17 11	75 30	112·8	4·66	2	5·0 6·1	·534	W.B.
Hu. 488 ...	17 11	69 58	105·0	2·73	3	8·8 10·0	·411	W.B.
Hu. 489 ...	17 12	69 47	45·9	0·97	3	9·2 10·5	·420	W.B.

Micrometric Observations of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Distance.	No. of Nights.	Mags.		Epoch 1912.	Obs.
	h m	° ′	°	″					
OΣ 327 ...	17 12	33 46	144·3	0·39	1	7·6	7·9	·769	B.
HCW 15 ...	17 13	63 19	44·4	0·50	3	8·3	9·3	·423	W.B.
Ait. 232 ...	17 18	64 12	95·9	0·56	1	8·7	9·4	·375	W.B.
Σ 2157 ...	17 19	73 27	206·0	3·38	2	8·3	9·7	·457	W.B.
β 46 ...	17 20	76 31	204·3	1·85	3	7·7	10·9	·439	W.B.
Hu. 1179 ...	17 21	51 2	277·0	0·24	3	7·0	7·1	·689	B.
Σ 2165 ...	17 23	60 28	56·0	7·61	1	7·0	8·5	·490	H.F.
Σ 2168 ...	17 23	54 11	17·8	1·86	1	7·5	8·2	·772	W.B.
Σ 2170 ...	17 24	79 27	68·7	3·48	1	8·5	9·0	·531	W.B.
Σ 2177 ...	17 25	43 31	135·0	3·46	1	8·5	10·0	·772	W.B.
Σ 2173 ...	17 26	91 0	185·3	0·25	3	6·3	6·3	·656	B.
Hu. 179 ...	17 27	78 43	53·2	1·99	1	8·8	8·9	·528	H.F.
Σ 2182 ...	17 29	66 4	359 1	5·33	2	8·2	9·2	·457	W.B.
			359·9	5·37	2	...		·509	H.F.
β 1121 ...	17 33	77 25	238·1	0·85	2	8·5	9·0	·500	H.F.
AG 210 ...	17 34	66 59	175·0	4·27	2	9·0	9·3	·491	H.F.
			173·7	4·07	2	...		·534	W.B.
β 631 ...	17 35	90 36	56·5	0·28	3	7·0	7·0	·656	B.
Σ 2199 ...	17 37	34 12	84·5	1·54	1	7·2	7·8	·772	W.B.
AG 211 ...	17 37	69 40	129·0	2·66	2	9·0	9·5	·481	H.F.
Σ 2203 ...	17 38	48 18	320·7	0·80	1	7·5	7·8	·441	S.C.
			315·1	0·69	2	...		·531	W.B.
Ho. 560 ...	17 40	56 1	89·3	0·80	1	8·0	8·0	·441	S.C.
Σ 2205 ...	17 42	72 15	310·4	1·99	2	8·3	8·7	·459	W.B.
			312·2	2·23	2	...		·481	H.F.
Σ 2215 ...	17 43	72 17	285·9	0·74	3	5·9	7·9	·475	W.B.
			283·7	0·72	2	...		·481	H.F.
Σ 2217 ...	17 44	75 13	102·8	6·81	2	7·6	8·0	·553	W.B.
β 824 ...	17 44	19 51	354·1	0·78	2	8·5	8·6	·616	B.
OΣ 337 ...	17 46	82 45	266·8	0·30	3	7·5	8·0	·562	B.
Σ 2238 ...	17 47	52 14	289·2	1·91	2	9·2	9·7	·655	W.B.
OΣ 338 ...	17 48	74 40	13·1	0·70	2	6·6	6·9	·448	W.B.
			16·0	0·97	1	...		·539	S.C.
			14·2	0·84	3	...		·562	B.
Ait. 234 ...	17 49	64 23	37·6	0·43	2	8·8	9·1	·627	B.
AC 8 ...	17 49	60 19	255·9	0·32	2	8·2	8·2	·627	B.
Ait. 235 ...	17 50	65 0	264·3	0·40	2	7·9	8·1	·627	B.

Micrometric Observations of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle	Distance.	No. of Nights.	Mags.	Epoch 1912.	Obs.
	h m	° ′	°	″				
AC 9 ...	17 51	60 10	235·8	1·04	1	8·3 8·8	·490	H.F.
			237·2	0·97	2	...	·529	W.B.
OΣ 339 ...	17 52	68 29	177·6	3·34	2	7·5 9·9	·445	W.B.
			176·8	3·18	3	...	·573	H.F.
β 1299 ...	17 53	79 2	159·0	0·54	2	8·5 8·5	·791	B.
β 1127 ...	18 0	45 46	125·0	0·90	1	7·8 9·7	·767	W.B.
Σ 2275 ...	18 0	50 39	136·8	0·63	2	9·0 9·2	·791	B.
Σ 2272 ...	18 1	87 27	145·9	3·79	13	4·5 6·0	·620	W.B.
OΣ 341 ...	18 2	68 34	94·1	0·59	1	6·4 7·7	·441	S.C.
			91·5	0·35	3	...	·773	B.
OΣ 524 ...	18 4	70 21	56·6	0·36	2	7·0 8·3	·786	B.
Σ 2281 ...	18 5	86 2	258·2	0·24	2	5·7 7·2	·775	B.
Σ 2283 ...	18 5	83 52	88·2	0·67	1	7·2 7·7	·737	B.
Σ 2285 ...	18 5	76 32	336·5	3·62	2	8·2 10·0	·481	H.F.
			332·2	3·34	2	...	·564	W.B.
Hu. 314 ...	18 5	71 23	138·3	0·57	2	8·3 8·5	·564	W.B.
Hu. 316 ...	18 6	71 45	160·0	1·69	2	9·0 10·1	·481	H.F.
			153·9	1·68	3	...	·501	W.B.
Σ 2289 ...	18 6	73 33	228·5	1·26	2	6·0 7·1	·459	W.B.
			230·2	1·19	2	...	·481	H.F.
			227·6	1·43	1	...	·539	S.C.
Hu. 317 ...	18 7	72 48	22·7	1·62	2	8·5 8·8	·459	W.B.
			18·0	2·10	2	...	·481	H.F.
Ait. 238 ...	18 8	64 42	97·2	0·48	2	8·5 8·8	·497	W.B.
Σ 2292 ...	18 9	62 23	266·1	1·06	2	8·0 8·1	·444	W.B.
			269·4	1·36	1	...	·539	S.C.
Σ 2294 ...	18 10	89 51	287·8	0·36	1	7·4 7·7	·737	B.
OΣ 346 ...	18 12	70 16	331·0	5·36	1	7·5 8·3	·739	W.B.
AG 221 ...	18 16	68 42	14·5	1·34	1	9·0 9 4	·471	H.F.
			11·4	1·29	2	...	·476	W.B.
Ho. 430 ...	18 18	69 32	194·8	2·45	2	8·5 8·9	·477	W.B.
AG 222 ...	18 18	75 49	146·8	2·06	1	8·6 8·8	·539	S.C.
β 641 ...	18 18	68 32	341·9	1·05	1	7·1 9·0	·471	H.F.
			341·7	0·94	2	...	·540	W.B.
Ho. 83 ...	18 20	62 32	83·6	0·57	2	8·8 8·8	·644	W.B.
Σ 2314 ...	18 20	66 35	327·4	2·24	4	8·4 9·6	·497	W.B.
Σ 2315 ...	18 21	62 39	181·9	0·26	3	7·0 8·0	·705	W.B.
Hu. 244 ...	18 27	78 2	257·6	0·94	1	8·9 12·2	·467	W.B.

Micrometric Observations of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Distance.	No. of Nights.	Mags.		Epoch 1912.	Obs.
	h m	° ′	°	″					
Hu. 245 ...	18 29	78 17	52·0	2·03	3	8·2	9·2	·566	W.B.
Hu. 322 AB	18 30	72 21	74·1	0·40	1	8·0	8·2	·586	W.B.
Σ 2339 AC	...		269·0	2·32	1	7·2	8·0	·586	W.B.
OΣ 357 ...	18 32	78 21	234·4	0·52	4	7·5	7·6	·506	W.B.
OΣ 359 ...	18 32	66 28	326·5	0·26	4	6·6	6·9	·604	W.B.
Hu. 247 ...	18 33	79 49	43·8	0·40	1	9·0	9·3	·467	W.B.
Σ 2356 ...	18 35	61 23	56·5	0·98	2	8·0	9·0	·480	W.B.
β 265 ...	18 46	78 35	227·1	1·21	1	7·1	9·1	·542	W.B.
Σ 2415 ...	18 51	69 30	291·4	1·75	2	6·6	8·5	·445	W.B.
Σ 2422 ...	18 53	64 2	90·3	0·84	2	7·6	7·7	·444	W.B.
β 648 ...	18 54	57 13	183·8	0·42	1	6·0	9·5	·761	W.B.
Ait. 260 ...	18 54	57 58	244·9	1·02	1	8·9	9·1	·761	W.B.
Hu. 330 ...	18 55	70 32	34·6	0·79	1	9·0	9·3	·467	W.B.
β 649 ...	18 56	57 40	9·4	1·60	1	8·5	11·7	·761	W.B.
β 466 ...	19 1	99 18	164·0	1·73	2	9·2	10·0	·504	W.B.
Ho. 95 ...	19 2	62 51	217·6	0·34	1	8·0	8·0	·761	W.B.
Σ 2454 ...	19 3	59 42	254·3	0·77	2	8·0	9·2	·797	W.B.
Σ 2455 ...	19 3	67 58	71·8	3·64	2	7·2	8·3	·794	W.B.
Σ 2457 ...	19 3	67 33	202·3	10·59	1	8·2	8·7	·756	W.B.
Σ 2466 ...	19 4	60 20	105·9	2·08	2	8·0	8·5	·759	W.B.
*	19 5	71 4	134·8	12·01	1	10·0	10·0	·756	W.B.
Ho. 442 ...	19 5	70 55	93·3	4·17	1	9·0	10·5	·542	W.B.
Σ 2481 ...	19 8	51 22	218·4	4·35	2	8·0	8·0	·759	W.B.
Σ 2482 ...	19 9	71 1	344·9	1·88	2	8·5	9·2	·480	W.B.
β 139 ...	19 9	73 18	139·9	0·70	1	6·7	8·0	·739	W.B.
OΣ 368 ...	19 12	74 0	213·8	0·95	4	7·3	8·5	·626	W.B.
			215·4	0·89	1	...		·758	H.F.
OΣ 371 ...	19 12	62 42	158·5	0·80	2	6·8	6·9	·786	W.B.
Σ 2491 ...	19 13	61 53	217·8	1·10	2	7·9	9·2	·575	W.B.
Hu. 336 ...	19 13	71 17	193·4	1·29	1	8·9	9·2	·542	W.B.
Espin 80 ...	19 21	57 2	187·9	3·53	2	8·6	9·0	·759	W.B.
β 1129 ...	19 21	37 48	328·0	0·36	1	6·3	6·3	·769	B.
Σ 3111 ...	19 21	68 21	116·6	2·47	2	9·0	9·4	·518	W.B.
Σ 2524 ...	19 23	64 42	98·9	6·56	2	8·3	8·5	·757	W.B.
Σ 2525 ...	19 23	62 52	305·0	0·69	3	7·5	7·9	·524	W.B.
			307·9	0·66	1	...		·758	H.F.
Ho. 450 ...	19 23	51 22	275·9	0·89	3	8·0	8·7	·789	W.B.
β 424 ...	19 24	54 7	38·8	2·95	2	8·7	10·1	·794	W.B.

Micrometric Observations of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910	Position Angle.	Distance.	No. of Nights.	Mags.		Epoch 1912.	Obs.
	h m	° ′	°	″					
Ait. 160 ...	19 25	67 6	42·9	0·50	1	8·5	8·6	·758	H.F.
β 143 ...	19 28	40 41	191·3	2·16	2	8·0	9·1	·803	W.B.
Σ 2540 ...	19 29	69 47	146·8	5·25	1	7·5	9·0	·758	H.F.
OΣ 375 ...	19 31	72 4	151·5	0·61	4	7·2	8·4	·631	W.B.
AG 234 ...	19 32	53 55	326·0	2·49	2	9·3	9·4	·794	W.B.
Σ 2556 ...	19 36	67 57	135·0	0·41	3	7·3	7·3	·692	W.B.
			126·8	0·45	2	...		·753	B.
			139·9	0·53	1	...		·758	H.F.
Ait. 166 ...	19 36	66 42	239·7	0·62	1	9·0	9·1	·758	H.F.
OΣ 380 ...	19 38	78 23	74·5	0·45	4	6·0	7·2	·631	W.B.
			76·0	0·52	2	...		·753	B.
			73·4	0·54	1	...		·758	H.F.
OΣ 382 ..	19 38	62 50	134·9	0·38	2	7·1	7·6	·753	B.
Σ 2569 ...	19 41	73 24	2·6	2·25	1	8·0	8·5	·758	H.F.
Σ 2576 ...	19 42	56 36	286·3	1·98	2	7·8	7·8	·764	W.B.
Σ 2583 ...	19 44	78 25	114·3	1·26	2	6·0	6·7	·529	W.B.
			115·6	1·40	1	...		·758	H.F.
OΣ 387 ...	19 45	54 56	321·7	0·58	2	7·2	8·2	·767	W.B.
β 978 ...	19 48	66 43	232·9	0·91	2	8·3	8·4	·540	W.B.
OΣ 388 ...	19 49	64 22	142·6	3·53	2	7·6	7·6	·794	W.B.
Σ 2596 ...	19 50	74 56	328·9	1·89	2	7·2	8·6	·529	W.B.
OΣ 395 ...	19 58	65 19	104·4	0·65	2	5·8	6·2	·753	B.
Σ 2653 ...	20 10	66 3	264·2	2·56	1	7·0	10·1	·728	W.B.
Espin 27 ...	20 11	43 25	338·8	3·77	2	9·4	9·5	·794	W.B.
β 984 ...	20 14	63 55	216·3	0·73	2	7·9	8·2	·529	W.B.
Σ 2672 ...	20 18	66 31	298·7	0·82	3	8·7	8·8	·684	W.B.
De. 22 ...	20 22	50 12	145·9	2·65	2	7·9	9·0	·764	W.B.
Σ 2695 ...	20 28	64 30	77·0	1·03	3	6·8	8·0	·626	W.B.
			78·4	0·98	1	...		·578	S.C.
β 151 ...	20 33	75 43	223·9	0·25	3	4·5	6·0	·626	W.B.
			247·3	0·19	4	...		·797	B.
Σ 2716 ...	20 37	58 1	48·5	2·81	1	6·0	8·2	·578	S.C.
β 364 ...	20 43	64 55	228·0	1·22	2	8·7	8·9	·646	W.B.
β 66 ...	20 44	62 52	161·0	1·07	2	8·6	9·1	·767	W.B.
Ho. 144 ...	20 48	70 13	166·2	0·31	2	7·0	7·0	·657	W.B.
OΣ 417 ...	20 49	61 11	28·0	0·65	3	7·5	8·1	·684	W.B.
			29·5	0·72	1	...		·758	H.F.

Micrometric Observations of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Distance.	No. of Nights.	Mag.		Epoch 1912.	Obs.
	h m	° ′	°	″					
OΣ 418 ...	20 51	57 38	292·6	1·37	1	7·3	7·4	·578	S.C.
			288·2	1·14	2	...		·767	W.B.
Σ 2737 ...	20 55	86 3	286·9	0·29	2	5·7	6·2	·753	B.
β 69 ...	20 59	68 40	312·5	0·85	2	8·2	9·0	·575	W.B.
			316·4	0·81	1	...		·758	H.F.
OΣ 430 ...	21 8	66 13	199·9	1·39	1	7·8	9·8	·832	W.B.
Hu. 767 ...	21 9	74 24	135·8	0·24	3	7·0	7·0	·791	B.
Ho. 152 ...	21 9	62 2	323·1	0·46	1	8·4	8·5	·761	W.B.
OΣ 535 ...	21 10	80 22	251·7	0·14	3	4·5	5·0	·791	B.
OΣ 432 ...	21 11	49 14	124·8	1·29	2	6·8	7·2	·802	W.B.
			126·0	1·28	1	...		·830	S.C.
β 162 ...	21 13	54 37	247·7	1·19	2	8·0	8·5	·796	W.B.
Ho. 153 ...	21 14	58 37	125·9	0·78	1	8·0	9·0	·832	W.B.
Ho. 286 ...	21 16	52 8	6·8	0·18	1	6·0	6·0	·868	B.
Ait. 617 ...	21 17	80 2	278·3	0·19	2	7·0	7·0	·803	B.
OΣ 437 ...	21 17	57 55	41·4	1·80	1	6·5	7·2	·832	W.B.
			39·1	1·96	2	...		·704	S.C.
β 164 ...	21 21	81 0	234·4	0·47	1	7·5	8·0	·586	W.B.
Holden 45	21 27	55 20	191·7	1·35	1	8·5	9·1	·832	W.B.
Hu. 371 ...	21 31	65 57	158·4	0·22	2	7·0	7·5	·819	B.
β 1212 ...	21 35	90 28	288·2	0·53	2	6·5	6·9	·803	B.
OΣ 448 ...	21 37	61 4	230·4	0·72	3	7·7	8·7	·719	W.B.
Ho. 164 ...	21 37	55 20	65·4	3·60	1	8·0	8·0	·832	W.B.
Σ 2822 ...	21 40	61 40	126·7	1·56	1	4·0	5·0	·832	W.B.
			130·8	1·98	2	...		·704	S.C.
β 989 ...	21 41	64 46	238·9	0·20	2	4·3	5·0	·575	W.B.
			255·0	0·20	3	...		·791	B.
Ho. 168 ...	21 42	46 29	250·8	1·06	1	8·2	8·2	·832	W.B.
Ho. 171 ...	21 48	62 38	170·9	0·81	1	8·2	8·2	·761	W.B.
β 696 ...	22 0	74 34	359·9	0·57	1	8·0	8·0	·564	W.B.
Σ 2881 ...	22 10	60 53	99·2	1·77	1	7·7	8·2	·578	S.C.
			100·2	1·53	3	...		·788	W.B.
			89·3	1·69	1	...		·857	H.F.
AG 281 ...	22 11	68 30	21·5	2·53	1	8·8	9·5	·857	H.F.
β 172 ...	22 19	95 18	357·5	0·61	1	6·7	6·7	·737	B.
Σ 2900 ...	22 19	69 36	173·5	1·26	3	6·0	9·2	·853	W.B.
Σ 2909 ...	22 24	90 29	309·9	2·53	1	4·0	4·1	·800	W.B.

Micrometric Observations of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Distance.	No. of Nights.	Mags.		Epoch 1912.	Obs.
	h m	° ′	°	″					
Ho. 475 AB	22 30	64 3	314·4	0·88	2	8·0	8·2	·879	W.B.
AC			226·8	8·36	1	8·0	10·2	·969	W.B.
Σ 2934 ...	22 38	69 3	126·8	0·83	2	8·2	9·2	·879	W.B.
Ho. 296 ...	22 36	75 56	50·9	0·40	1	5·5	5·5	·737	B.
OΣ 483 ...	22 55	78 45	233·3	0·73	2	6·2	7·7	·677	W.B.
Σ 2989 ...	23 9	70 30	138·4	1·54	2	8·5	9·9	·879	W.B.
Σ 2990 ...	23 9	68 25	244·8	1·66	2	8·0	8·8	·879	W.B.
Σ 3020 ...	23 27	71 43	108·4	1·87	2	7·7	9·7	·879	W.B.
Σ 3033 ...	23 39	83 15	3·9	3·05	1	8·5	8·5	·789	W.B.
AGC 14 ...	23 39	61 8	198·5	1·25	2	5·0	8·1	·879	W.B.
Σ 3050 ...	23 55	56 46	221·9	2·24	1	6·0	6·0	·578	S.C.